Exercise 4

A Solving Polynomial Inequalities

1
Solution

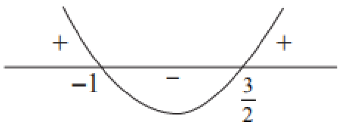
(a) $2x(x-1) \le 3-x$

$2x^2 - 2x \le 3-x$

$2x^2 - x - 3 \le 0$

$(2x-3)(x+1) \le 0$

$\therefore \ -1 \le x \le \dfrac{3}{2}$



Learning point:
Since $(2x-3)(x+1) \le 0$ (the inequality sign is less than zero), the region below the x-axis satisfy the inequality. (See above diagram)

(b) $x(x-2) > 5$

$x^2 - 2x - 5 > 0$

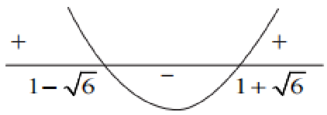
Let $x^2 - 2x - 5 = 0$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(-5)}}{2(1)}$$

$x = \dfrac{2+\sqrt{24}}{2}$ or $x = \dfrac{2-\sqrt{24}}{2}$

$x = 1+\sqrt{6}$ or $x = 1-\sqrt{6}$

$\therefore \ x < 1-\sqrt{6}$ or $x > 1+\sqrt{6}$



Learning point:
Since $x^2 - 2x - 5 > 0$ (the inequality sign is more than zero), the region above the x-axis 、 satisfy the inequality. (See above diagram)

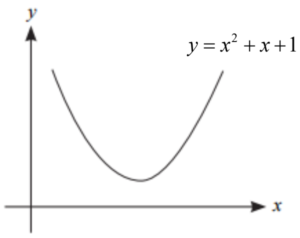
(c) $x^2 + x + 1 > 0$

$$\left(x+\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 1 > 0$$

$$\left(x+\frac{1}{2}\right)^2 + \frac{3}{4} > 0$$

$\because \ \left(x+\dfrac{1}{2}\right)^2 \ge 0, \ x \in \mathbb{R}.$

$\therefore \ \left(x+\dfrac{1}{2}\right)^{22} + \dfrac{3}{4}$ is always positive.



$x \in \mathbb{R}$

(d) $x^2 - 3x + 5 \le 0$

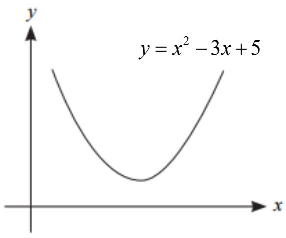
$\left(x - \frac{3}{2}\right)^2 - \frac{9}{4} + 5 \le 0$ ◁ completing the square

$\left(x - \frac{3}{2}\right)^2 + \frac{11}{4} \le 0$

Since $\left(x - \frac{3}{2}\right)^2 \ge 0$ for all real values of x

$\therefore$ $\left(x - \frac{3}{2}\right)^2 + \frac{11}{4}$ is always positive.

$\therefore$ there is no solution for x.

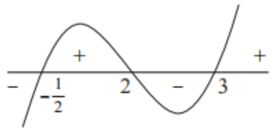


2
Solution

(a) $(x-3)(2x+1)(2-x) \le 0$ ◁ multiply (–) on both sides

$(x-3)(2x+1)(x-2) \ge 0$

$\therefore\ -\dfrac{1}{2} \le x \le 2$ or $x \ge 3$



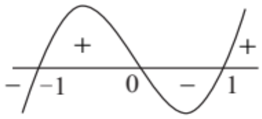
(b) $x^3 > x$

$x^3 - x > 0$

$x(x^2-1) > 0$

$x(x+1)(x-1) > 0$

$\therefore\ -1 < x < 0$ or $x > 1$



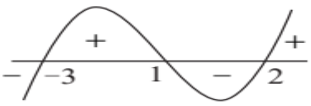
(c) $x^3 \ge 7x - 6$

$x^3 - 7x + 6 \ge 0$ ◁ factorise

$(x-1)(x^2+x-6) \ge 0$

$(x-1)(x+3)(x-2) \ge 0$

$\therefore\ -3 \le x \le 1$ or $x \ge 2$



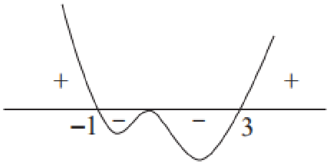
3
Solution

(a) $x^4 - 2x^3 - 3x^2 \geq 0$

$x^2(x^2 - 2x - 3) \geq 0$

$(x-0)^2(x+1)(x-3) \geq 0$

$\therefore\ x \geq 3$ or $x \leq -1$ or $x = 0$

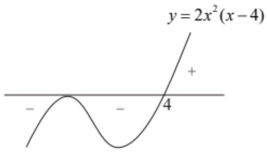


(b) $2x^2(x-4) > 0$

Since $x^2 \geq 0$ for all $x \in \mathbb{R}$

$\therefore\ x - 4 > 0$

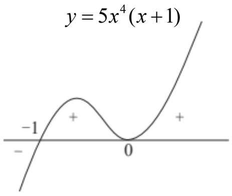
Hence $x > 4$



(c) $5x^4(x+1) \geq 0$

$x^4(x+1) \geq 0$

$\therefore\ x \geq -1$ or $x = 0$



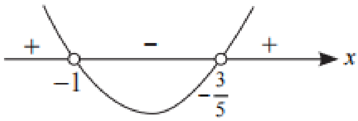
4
Solution

$$4x^2 + 16xy + y^2 + 16x + 14y + 13 = 0$$
$$y^2 + (16x + 14)y + (14x^2 + 16x + 3) = 0$$

Given that y has 2 distinct real values.
Consider $b^2 - 4ac > 0$

$$(16x + 4)^2 - 4(4x^2 + 16x + 13) > 0$$
$$64x^2 + 112x + 49 - 4x^2 - 16x - 13 > 0$$
$$5x^2 + 8x + 3 > 0 \quad \text{(Shown)}$$

$$5x^2 + 8x + 3 > 0$$
$$(5x + 3)(x + 1) > 0$$
$$\therefore \left\{ x : x \in \mathbb{R},\ x < -1 \text{ or } x > -\frac{3}{5} \right\}$$



5
Solution

$$y = \frac{x^2 - x - 1}{x + 4}$$

$$x^2 - x - 1 = xy + 4y$$

$$x^2 + (-1 - y)x + (-1 - 4y) = 0 \text{ .... (1)}$$

For the quadratic equation (1) not to have real roots,

$$\text{discriminant} < 0$$

$$(-1 - y)^2 - 4(-1 - 4y) < 0$$

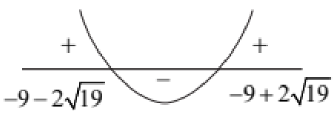
$$y^2 + 2y + 1 + 4 + 16y < 0$$

$$y^2 + 18y + 5 < 0$$

$$(y + 9)^2 + 5 - 81 < 0$$

$$(y + 9 + \sqrt{76})(y + 9 - \sqrt{76}) < 0$$

$$\therefore \ \{x : x \in \mathbb{R}, \ -9 - 2\sqrt{19} < y < -9 + 2\sqrt{19} \}$$



Alternative Method

Let $y^2 + 18y + 5 = 0$

$$y = \frac{-18 \pm \sqrt{18^2 - 4(5)}}{2}$$

$$= \frac{-18 \pm \sqrt{304}}{2}$$

$$= \frac{-18 \pm 4\sqrt{19}}{2}$$

$$= -9 \pm 2\sqrt{19}$$

$$\therefore \ \{x : x \in \mathbb{R}, \ -9 - 2\sqrt{19} < y < -9 + 2\sqrt{19} \}$$

Exercise 4

B Solving Inequalities involving Rational Functions

6
Solution

(a) Given $\dfrac{9x+2}{x-2} > 2$

$$\frac{9x+2}{x-2} - 2 > 0$$

$$\frac{9x+2-2(x-2)}{x-2} > 0$$

$$\frac{9x+2-2x+4}{x-2} > 0$$

$$\frac{7x+6}{x-2} > 0$$

$$(7x+6)(x-2) > 0$$

$\therefore\ x < -\dfrac{6}{7}$ or $x > 2$

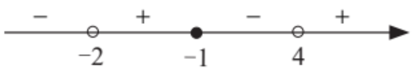
(b) $\dfrac{2}{x+2} \geq \dfrac{x-9}{x^2-2x-8}$

$$\frac{2}{x+2} - \frac{x-9}{(x+2)(x-4)} \geq 0$$

$$\frac{2(x-4)}{(x+2)(x-4)} - \frac{x-9}{(x+2)(x-4)} \geq 0$$

$$\frac{2(x-4)-(x-9)}{(x+2)(x-4)} \geq 0$$

$$\frac{x+1}{(x+2)(x-4)} \geq 0$$



$\therefore\ -2 < x \leq -1$ or $x > 4$

(c) $2 < \dfrac{x-1}{x+1} \le 4$

$2 < \dfrac{x-1}{x+1}$

$\dfrac{x-1}{x+1} > 2$

$\dfrac{x-1}{x+1} - 2 > 0$

$\dfrac{x-1-2x-2}{x+1} > 0$

$\dfrac{x+3}{x+1} < 0$

$(x+3)(x+1) < 0$

$-3 < x < -1$

and

$\dfrac{x-1}{x+1} \le 4$

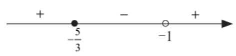
$\dfrac{x-1}{x+1} - 4 \le 0$

$\dfrac{x-1-4x-4}{x+1} \le 0$

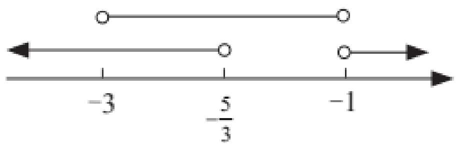
$\dfrac{-3x-5}{x+1} \le 0$

$\dfrac{3x+5}{x+1} \ge 0$

$(3x+2)(x+1) \ge 0$



$x \le -\dfrac{5}{3}$ or $x > -1$



$\therefore\ -3 < x < -\dfrac{5}{3}$

Learning point:

The overlapping lines is the solution set that satisfies the original inequality.

(d) $$2x+\frac{1}{1+2x}>0$$

$$\frac{4x^2+2x+1}{1+2x}>0$$

$(4x^2+2x+1)(1+2x)>0$ ◁ completing the square, $4x^2+2x+1$

$$\left[4\left(x+\frac{1}{2}\right)^2+\frac{3}{4}\right](1+2x)>0$$

Since $4\left(x+\frac{1}{4}\right)^2 \geq 0,\ x\in\mathbb{R}\ \therefore\ 4\left(x+\frac{1}{4}\right)^2+\frac{1}{4}$ is always positive.

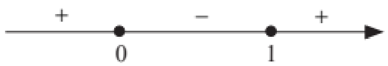
$\therefore\ x>-\frac{1}{2}$

(e) $$\frac{x+1}{x^2+1}\geq 1$$

$x+1\geq x^2+1$ ◁ collecting all the terms to one side

$x(x-1)\leq 0$

$\therefore\ 0\leq x\leq 1$



(h) $$\frac{x^2+x+1}{x^2+2}>0$$

$$(x^2+x+1)(x^2+2)>0$$

$$\left[\left(x+\frac{1}{2}\right)^2+\frac{3}{4}\right](x^2+2)>0$$

As $\left(x+\frac{1}{2}\right)^2\geq 0$ for $x\in\mathbb{R}$, so $\left(x+\frac{1}{2}\right)^2+\frac{3}{4}$ is always positive.

As $x^2\geq 0$ for $x\in\mathbb{R}$, so x^2+1 are always positive.

$\therefore\ x\in\mathbb{R}$

7
Solution

$$\frac{x^3+x^2-8x-12}{x^3+4x^2+5x} \le 0$$

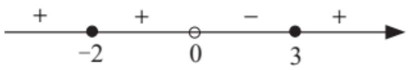
$$\frac{(x-3)(x+2)^2}{x(x^2+4x+5)} \le 0 \qquad \triangleleft \text{ factorise}$$

$$\frac{(x-3)(x+2)^2}{x\left[(x+2)^2+1\right]} \le 0$$

Since $x^2+4x+5=(x+2)^2+1>0 \ \forall \ x \in \mathbb{R}$

$$\frac{(x-3)(x+2)^2}{x} \le 0$$

$x(x-3)(x+2)^2 \le 0$



$\therefore \ x=-2 \quad \text{or} \quad 0<x\le 3$

8
Solution

(a) $\dfrac{x+8}{x^2-3x-4} \le -1$ .... (1)

$1+\dfrac{x+8}{x^2-3x-4} \le 0$

$\dfrac{x^2-2x+4}{x^2-3x-4} \le 0$ ◁ completing the square, x^2-2x+4

$\dfrac{(x-1)^2+3}{(x+1)(x-4)} \le 0$

$[(x-1)^2+3](x+1)(x-4) \le 0$

$(x-1)^2+3$ is always positive for real values of x

$\therefore\ (x+1)(x-4) \le 0$

Hence $-1 < x < 4$

(b) Replace x with $-x$ in (1):

$\dfrac{-x+8}{x^2+3x-4} \le -1$

$\dfrac{x-8}{x^2+3x-4} \ge 1$

However, we are solving $\dfrac{x-8}{x^2+3x-4} \le 1.$

Hence, deducing from part **(a)**'s result : $-x < -1$ or $-x > 4$

$\therefore\ x < -4$ or $x > 1$

9
Solution

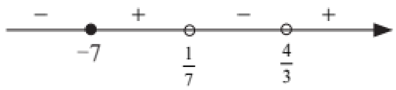
Given $\quad \dfrac{2}{7x-1} \geq \dfrac{1}{3x-4}$ .... (1)

$$\frac{-7-x}{(7x-1)(3x-4)} \geq 0$$

$$\frac{x+7}{(7x-1)(3x-4)} \leq 0$$

$$(x+7)(7x-1)(3x-4) \leq 0$$

$\therefore\ x \leq -7$ or $\dfrac{1}{7} < x < \dfrac{4}{3}$ .... (*)



Replace x with x^2 in (1): $\dfrac{2}{7x^2-1} \geq \dfrac{1}{3x^2-4}$

Deducing (*)'s result:

$x^2 \leq -7$ (No solution since $x^2 \geq 0$ for all real x)) or $\dfrac{1}{7} < x^2 < \dfrac{4}{3}$

Now consider $\dfrac{1}{7} < x^2 < \dfrac{4}{3}$.

$\dfrac{1}{7} < x^2$ and $x^2 < \dfrac{4}{3}$

For $\dfrac{1}{7} < x^2$

$$0 < x^2 - \frac{1}{7}$$

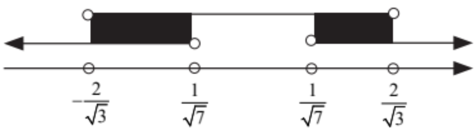
$\therefore \quad x > \dfrac{1}{\sqrt{7}}$ or $x < -\dfrac{1}{\sqrt{7}}$

For $x^2 < \dfrac{4}{3}$.

$$x^2 - \frac{4}{3} < 0$$

$$\left(x - \sqrt{\frac{4}{3}}\right)\left(x + \sqrt{\frac{4}{3}}\right) < 0$$

$\therefore -\sqrt{\dfrac{4}{3}} < x < \sqrt{\dfrac{4}{3}}$



$\therefore \quad -\dfrac{2}{\sqrt{3}} < x < -\dfrac{1}{\sqrt{7}}$ or $\dfrac{1}{\sqrt{7}} < x < \dfrac{2}{\sqrt{3}}$

10
Solution

(a) Given $\dfrac{x+4}{3+2x-x^2}<1$ .... (1)

$$\frac{x+4}{3+2x-x^2}-1<0$$

$$\frac{x+4}{3+2x-x^2}-\frac{3+2x-x^2}{3+2x-x^2}<0$$

$$\frac{x^2-x+1}{3+2x-x^2}<0$$

$$\frac{x^2-x+1}{x^2-2x-3}>0$$

Since $x^2-x+1=\left(x-\dfrac{1}{2}\right)^2+\dfrac{3}{4}>0$ for any real x

$\therefore$ $$\frac{1}{x^2-2x-3}>0$$

$$x^2-2x-3>0$$

$$(x+1)(x-3)>0$$

$\therefore$ $x<-1$ or $x>3$ .... (*)

(b) To solve the inequality $\dfrac{x^2-4}{x^4+2x^2-3}<1$

Replacing x with $-x^2$ in (1): $\dfrac{(-x^2)+4}{3+2(-x^2)-(-x^2)^2}<1$

$\therefore$ $$\frac{-x^2+4}{-x^4-2x^2+3}<1$$

$$\frac{x^2-4}{x^4+2x^2-3}<1$$

Deduce the result in (*). Replace x with $(-x^2)$ in (*).

$-x^2<-1$ or $-x^2>3$ (No solution since $-x^2\le 0$ for all real x)

$x^2>1$

$\therefore$ $x<-1$ or $x>1$

11
Solution

Given $\dfrac{x+2}{2x-1} < 2x+1$ .... (1)

$$\frac{x+2}{2x-1} - (2x+1) < 0$$

$$\frac{x+2-(2x+1)(2x-1)}{2x-1} < 0$$

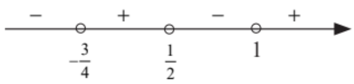
$$\frac{x+2-(4x^2-1)}{2x-1} < 0$$

$$\frac{-4x^2+x+3}{2x-1} < 0$$

$$\frac{4x^2-x-3}{2x-1} < 0$$ ◁ multiply (−) both sides and change the inequality sign

$$(2x-1)(4x^2-x-3) > 0$$

$$(2x-1)(4x+3)(x-1) > 0$$



$\therefore\ -\dfrac{3}{4} < x < \dfrac{1}{2}$ or $x > 1$ .... (*)

To solve the inequality $\dfrac{2x^2+1}{2-x^2} < \dfrac{2+x^2}{x^2}$

Replacing x with $\dfrac{1}{x^2}$ in (1):

$$\frac{2+\frac{1}{x^2}}{\frac{2}{x^2}-1} < \frac{2}{x^2}+1$$

$$\therefore \quad \frac{2x^2+1}{2-x^2} < \frac{2+x^2}{x^2}$$

Deduce the result in (*). Replace x by $\dfrac{1}{x^2}$ in (*).

$-\dfrac{3}{4} < \dfrac{1}{x^2} < \dfrac{1}{2}$ or $\dfrac{1}{x^2} > 1$

$-\dfrac{3}{4} < \dfrac{1}{x^2}$ and $\dfrac{1}{x^2} < \dfrac{1}{2}$ or $x^2 < 1$

$x \in \mathbb{R}$ $\quad 2 < x^2$ or $\dfrac{1}{x^2} > 1$

$x^2 > 2$ or $x^2 < 1$

$x^2 - 2 > 0$ or $x^2 - 1 < 0$

$(x-\sqrt{2})(x+\sqrt{2}) > 0$ $\quad (x-1)(x+1) < 0$

$\therefore\ x > \sqrt{2}$ or $x < -\sqrt{2}$ or $-1 < x < 1,\ x \neq 0$

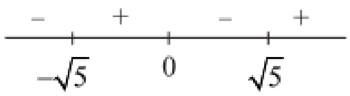
12
Solution

(a) $\dfrac{x}{x^2-5} \le 0$ .... (1)

$\dfrac{x}{(x-\sqrt{5})(x+\sqrt{5})} \le 0$

$x(x-\sqrt{5})(x+\sqrt{5}) \le 0$

$\therefore\ x < -\sqrt{5}$ or $0 \le x \le \sqrt{5}$ ....



(b) To solve the inequality $\dfrac{\sqrt{x}}{x-5} \le 0.$

Replace x by $\sqrt{x}$ in (1).

So, $\dfrac{\sqrt{x}}{(\sqrt{x})^2-5} \le 0$

Deduce the result in (*). Replace x by $\sqrt{x}$ in (*).

$\sqrt{x} < -\sqrt{5}$ (no solution since $\sqrt{x} \ge 0$) or $0 \le \sqrt{x} < \sqrt{5}$ ◁ square boths sides

$0 \le x < 5$

The set of values of x is $\{x \in \mathbb{R} :\ 0 \le x < 5\}$

13
Solution

Given $\quad \dfrac{x-10}{x+2} > x-5$ .... (1)

$$\frac{x-10}{x+2}-(x-5)>0$$

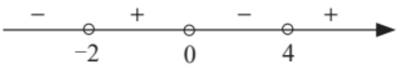
$$\frac{x-10-(x+2)(x-5)}{x+2}>0$$

$$\frac{x-10-(x^2-3x-10)}{x+2}>0$$

$$\frac{-x^2+4x}{x+2}>0$$

$$\frac{x(x-4)}{x+2}<0$$

$$x(x-4)(x+2)<0$$



$\therefore\ x<-2$ or $0<x<4$

To solve inequality $\dfrac{\sqrt{x}-11}{\sqrt{x}+1}>\sqrt{x}-6$.

Replace x by $\sqrt{x}-1$ in (1).

So, $\dfrac{\sqrt{x}-11}{\sqrt{x}+1}>\sqrt{x}-6$.

Deduce the result in (*). Replace x with $\sqrt{x}-1$ in (*).

Hence, $\quad 0<\sqrt{x}-1<4 \quad$ or $\quad \sqrt{x}-1<-2$

$0<\sqrt{x}-1<4$ or $\quad \sqrt{x}<-1$ (solution since $-x^2\le 0$ for all real x)

$1<\sqrt{x}<5 \quad \triangleleft$ square all sides

$1<x<5^2$

$\therefore\ 1<x<25$

14
Solution

(a) $2x^2 - 4x + 9$

$= 2\left(x^2 - 2x + \frac{9}{2}\right)$

$= 2(x-1)^2 + 7$

For all real values of x, $2(x-1)^2 \geq 0$.

So, $2(x-1)^2 + 7 > 0$

$2x^2 - 4x + 9 > 0$ is positive for all real values of x.

(b) $$\frac{5+2x}{x^2-3x+2} \geq -2 \quad \text{................. (1)}$$

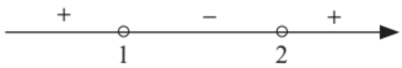
$$\frac{5+2x}{x^2-3x+2} + 2 \geq 0$$

$$\frac{5+2x+2(x^2-3x+2)}{x^2-3x+2} \geq 0$$

$$\frac{2x^2-4x+9}{(x-2)(x-1)} \geq 0, \quad x \neq 2 \text{ or } 1$$

In **(a)**, since $2x^2 - 4x + 9 > 0$

$\therefore \ (x-2)(x-1) > 0$



$x < 1$ or $x > 2$ .... (*)

(c) To solve inequality $\dfrac{5 - 2\ln x}{(\ln x)^2 + 3\ln x + 2} \geq -2$

Replace x with $-\ln x$ in (1).

So, $\dfrac{5 + 2(-\ln x)}{(-\ln x)^2 - 3[-\ln x] + 2} \geq -2$

Deduce the result in (*). Replace x with $-\ln x$ in (*).

$-\ln(x) < 1 \quad$ or $\quad -\ln(x) > 2$

$\ln(x) > -1 \quad$ or $\quad \ln(x) < -2$

$x > e^{-1} \quad$ or $\quad x < e^{-2}$

$\therefore \ 0 < x < e^{-2}$ or $x > e^{-1}$

Learning point:

It is incorrect to take $x < e^{-2}$

since $\ln x$ only exists, when $x > 0$.

$\therefore \ 0 < x < e^{-2}$

15
Solution

Given $\dfrac{x-3}{x^2+x-2} \le 1$ .... (1)

$\dfrac{x-3}{x^2+x-2} - 1 \le 0$

$\dfrac{x-3-(x^2+x-2)}{x^2+x-2} \le 0$

$\dfrac{-x^2-1}{(x-1)(x+2)} \le 0,\ x \ne -2,\ x \ne 1$

Since $-x^2-1 = -(x^2+1) < 0,$ for all real values of x.

Hence, $\dfrac{1}{(x-1)(x+2)} > 0,\ x \ne -2,\ x \ne 1$

$(x-1)(x+2) > 0$

$\therefore\ x < -2$ or $x > 1$ .... (*)

To solve inequality $\dfrac{e^{-x}-3}{e^{-2x}+e^{-x}-2} \le 1$

Replace x with e^{-x} in (1).

So $\dfrac{e^{-x}-3}{e^{-2x}+e^{-x}-2} \le 1$

Deduce the result in (*). Replace x by $\sqrt{x}$ in **(*)**

$e^{-x} < -2$ (no solution as $e^{-x} > 0$) or $e^{-x} > 1$

$\ln e^{-x} > \ln 1$ ◁ take In both sides

$-x > 0$

$x < 0$

$\therefore\ x < 0$

16
Solution

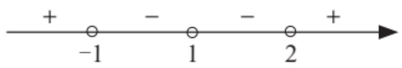
(a) $\dfrac{x^4-1}{(x-1)^3(x-2)}>0$

$\dfrac{(x-1)(x+1)(x^2+1)}{(x-1)^3(x-2)}>0$

$\dfrac{(x+1)(x^2+1)}{(x-1)^2(x-2)}>0$

Since $x^2+1>0\ \forall\, x\in\mathbb{R}$

$\dfrac{(x+1)}{(x-1)^2(x-2)}>0$



$(x+1)(x-1)^2(x-2)>0$

$\therefore\ x<-1$ or $x>2$ .... (*)

(b) To solve inequality $\dfrac{(\sqrt{x}-1)^3(\sqrt{x}-2)}{x^2-1}\geq 0$

$\dfrac{x^4-1}{(x-1)^3(x-2)}>0 \Leftrightarrow \dfrac{(x-1)^3(x-2)}{x^4-1}>0$

This means that $\dfrac{x^4-1}{(x-1)^3(x-2)}>0$ is true if and only if $\dfrac{(x-1)^3(x-2)}{x^4-1}>0$. .... (1)

Hence, replace x with $\sqrt{x}$ in (1).

So, $\dfrac{(\sqrt{x}-1)^3(\sqrt{x}-2)}{x^2-1}\geq 0$

$\dfrac{(\sqrt{x}-1)^3(\sqrt{x}-2)}{x^2-1}>0$ or $\dfrac{(\sqrt{x}-1)^3(\sqrt{x}-2)}{x^2-1}=0$

Consider $\dfrac{(\sqrt{x}-1)^3(\sqrt{x}-2)}{x^2-1}>0$

Replacing x with $\sqrt{x}$ in (*)

$\sqrt{x}<-1$ (no solution) or $\sqrt{x}>2$

Now consider $\dfrac{(\sqrt{x}-1)^3(\sqrt{x}-2)}{x^2-1}=0$

$\sqrt{x}=1$ (Rejected since $x\neq 1$) or $\sqrt{x}=2$

So, $\sqrt{x}\geq 2$

$\therefore\ x\geq 4$

Exercise 4

C Solving Modulus Functions

17
Solution

(a) $|x+2|=5$

$$x+2=5 \quad \text{or} \quad x+2=-5$$
$$x=3 \qquad\qquad x=-7$$

$\therefore\ x=3$ or $x=7$

(b) $|2x-6|=5$

$$2x-6=5 \quad \text{or} \quad 2x-6=-5$$
$$2x=11 \qquad\qquad 2x=1$$
$$x=\frac{11}{2} \qquad\qquad x=\frac{1}{2}$$

$\therefore\ x=\frac{11}{2}$ or $x=\frac{1}{2}$

(c) $|1-3x|=8$

$$1-3x=8 \quad \text{or} \quad 1-3x=-8$$
$$3x=-7 \qquad\qquad 3x=9$$
$$x=-\frac{7}{3} \qquad\qquad x=3$$

$\therefore\ x=-\frac{7}{3}$ or $x=3$

18
Soluton

(a) $|2x-5| = x$

$2x-5 = x$ or $2x-5 = -x$

$x = 5$ or $3x = 5$

$x = \frac{5}{3}$

Substituting $x = 5$ into $|2x-5| = x$.

L.H.S = R.H.S

Substituting $x = \frac{5}{3}$ into $|2x-5| = x$.

L.H.S = R.H.S

$\therefore$ $x = \frac{5}{3}$ or $x = 5$

(b) Let $|3-4x| = 6x$

$3-4x = 6x$ or $3-4x = -6x$

$3 = 10x$ $\quad$ $2x = -3$

$\therefore$ $x = \frac{3}{10}$ $\quad$ $x = -\frac{3}{2}$

Substituting $x = -\frac{3}{2}$ into $|3-4x| = 6x$

L.H.S = R.H.S

Substituting $x = \frac{3}{10}$ into $|3-4x| = 6x$.

L.H.S = R.H.S

$\therefore$ $x = \frac{5}{3}$ or $x = 5$

(c) Let $|2x-7| = 4x+1$

$2x-7 = 4x+1$ or $-(2x-7) = 4x+1$

$-8 = 2x$ $\quad$ $-2x+7 = 4x+1$

$\therefore$ $x = -4$ $\quad$ $6 = 6x$

$\therefore$ $x = 1$

Substituting $x = -4$ into $|2x-7| = 4x+1$.

L.H.S $\neq$ R.H.S

Substituting $x = 1$ into $|2x-7| = 4x+1$.

L.H.S = R.H.S

$\therefore$ $x = 1$

(d) $|3-2x| = 6x-5$

$3-2x = 6x-5$ or $3-2x = -(6x-5)$

$8 = 8x$ $\quad$ $3-2x = -6x+5$

$x = 1$ $\quad$ $-2 = -4x$

$-\frac{1}{2} = x$

Substituting $x = -\frac{1}{2}$ into $|3-2x| = 6x-5$.

L.H.S $\neq$ R.H.S

Substituting $x = 1$ into $|3-2x| = 6x-5$.

L.H.S = R.H.S

$\therefore\ x = 1$

19
Solution

(a) $\left|x^2-3\right|=1$

$x^2-3=1$ or $x^2-3=-1$

$x^2=4$ $\quad$ $x^2=2$

$x=2$ or -2 $\quad$ $x=\sqrt{2}$ or $-\sqrt{2}$

$\therefore\ x=\pm 2,\ \pm\sqrt{2}$

(b) $\left|x^2-4x+8\right|=2$

$x^2-7x+8=2$ or $x^2-7x+8=-2$

$x^2-7x+6=0$ $\quad$ $x^2-7x+10=0$

$(x-1)(x-6)=0$ $\quad$ $(x-2)(x-5)=0$

$x=1,\ 6$ $\quad$ $x=2,\ 5$

$\therefore\ x=1,\ 2,\ 5,\ 6$

(c) $\left|x^2-6\right|=5x$

$x^2-6=5x$ or $x^2-6=-5x$

$x^2-5x-6=0$ $\quad$ $x^2+5x-6=0$

$(x-6)(x+1)=0$ $\quad$ $(x+6)(x-1)=0$

$x=-1$ or 6 $\quad$ $x=1$ or -6

Substituting $x=-6$ into $\left|x^2-6\right|=5x$. L.H.S $\neq$ R.H.S

Substituting $x=-1$ into $\left|x^2-6\right|=5x$. L.H.S $\neq$ R.H.S

Substituting $x=1$ into $\left|x^2-6\right|=5x$. L.H.S $=$ R.H.S

Substituting $x=6$ into $\left|x^2-6\right|=5x$. L.H.S $=$ R.H.S

$\therefore\ x=1$ or 6.

(d) $\left| x^2 - 2x \right| = 2x - 3$

$x^2 - 2x = -(2x-3)$ or $x^2 - 2x = (2x-3)$

$x^2 - 2x = -2x + 3$ $\quad$ $x^2 - 4x + 3 = 0$

$x^2 = 3$ $\quad$ $(x-1)(x-3) = 0$

$x = \pm\sqrt{3}$ $\quad$ $x = 1,\ 3$

Substituting $x = -\sqrt{3}$ into $\left| x^2 - 2x \right| = 2x - 3$. L.H.S $\neq$ R.H.S

Substituting $x = \sqrt{3}$ into $\left| x^2 - 2x \right| = 2x - 3$. L.H.S = R.H.S

Substituting $x = 1$ into $\left| x^2 - 2x \right| = 2x - 3$. L.H.S $\neq$ R.H.S

Substituting $x = 3$ into $\left| x^2 - 2x \right| = 2x - 3$. L.H.S = R.H.S

$\therefore\ x = \sqrt{3}$ or 3.

(e) $|x(x-3)| = x - 3$

$|x^2 - 3x| = x - 3$

$x^2 - 3x = x - 3$ or $x^2 - 3x = -(x-3)$

$x^2 - 4x + 3 = 0$ $\quad$ $x^2 - 3x = -x + 3$

$(x-3)(x-1) = 0$ $\quad$ $x^2 - 2x - 3 = 0$

$x = 1,\ 3$ $\quad$ $(x-3)(x+1) = 0$

$x = 3$ or $x = -1$

Substituting $x = 1$ into $|x^2 - 3x| = x - 3$. L.H.S $\neq$ R.H.S

Substituting $x = -1$ into $|x^2 - 3x| = x - 3$. L.H.S $\neq$ R.H.S

Substituting $x = 3$ into $|x^2 - 3x| = x - 3$. L.H.S = R.H.S

$\therefore\ x = 3.$

(f) $\left| \frac{5}{2}x^2 + \frac{13}{2}x \right| = 10$ $\quad$ ⊲ multiply 2 on both sides

$|15x^2 + 13x| = 20$

$15x^2 + 13x = 20$ or $15x^2 + 13x = -20$

$15x^2 + 13x - 20 = 0$ $\quad$ $15x^2 + 13x + 20 = 0$

$(5x-4)(3x+5) = 0$ $\quad$ Using discriminant, $b^2 - 4ac$

$x = \frac{4}{5}$ or $-\frac{5}{3}$ $\quad$ $(13)^2 - 4(13)(20)$

$= -871 < 0$

$\therefore\ 15x^2 + 13x + 20 = 0$ has no real solution.

$\therefore\ x = \frac{4}{5}$ or $-\frac{5}{3}$

20
Solution

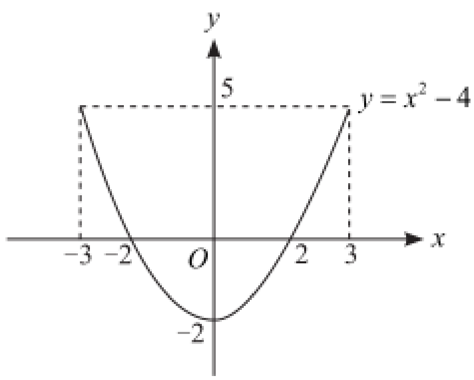
(a) Given $y = x^2 - 4$

Determine axial intercept
When $x = 0$, $y = 5$.
When $y = 0$, $x = 3$ or -3.

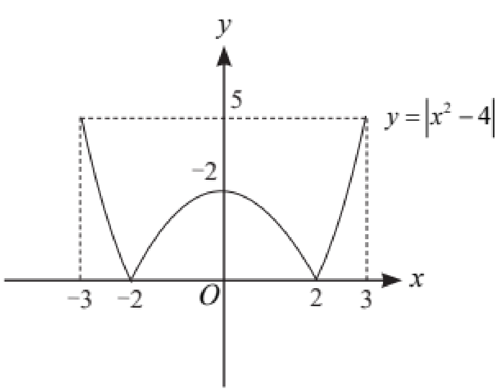
Determine the end-point of the graph
When $x = -3$, $y = 5$ and
when $x = 3$, $y = 5$.

Determine the turning of the graph
Using GC, the turning point is (0, 5)

Sketch $y = x^2 - 4$ for $-3 \le x \le 3$.



Then reflect the negative part of the graph in the x-axis $y = x^2 - 4$ for $-3 \le x \le 3$.



Learning point:
To obtain the graph $y = |\,\mathrm{f}(x)\,|$
• Retain the positive part of the graph, i.e. the graph above the x-axis.
• Reflect the negative part (i.e.the graph below the axis) of the graph $y = \mathrm{f}(x)$ in the x-axis.

(b) Given $y = 1 - \frac{1}{3}x^2$

Determine axial intercept

When $x = 0$, $y = 1$.

When $y = 0$, $x = \sqrt{3}$ or $-\sqrt{3}$.

Determine the end-point of the graph

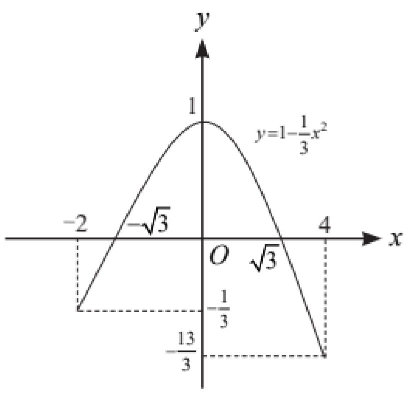
When $x = -2$, $y = -\frac{1}{3}$ and

when $x = 4$, $y = -\frac{13}{3}$.

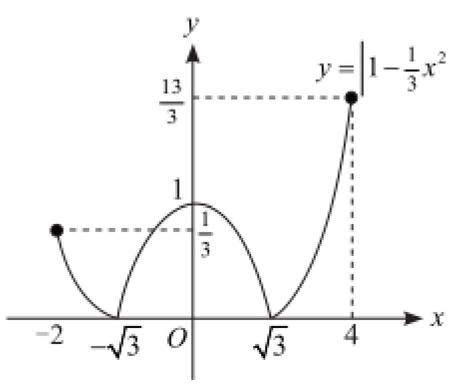
Determine the turning of the graph

Using GC, the turning point is (0, 1)

Sketch $y = 1 - \frac{1}{3}x^2$ for $-2 \le x \le 4$.



Reflect the negative part of the graph in the x-axis to obtain $y = \left|1 - \frac{1}{3}x^2\right|$.



(c) Given $y = (x-2)(x-3)$

Determine axial intercept

When $x = 0$, $y = 6$.

When $y = 0$, $x = 2$ or 3.

Determine the end-point of the graph

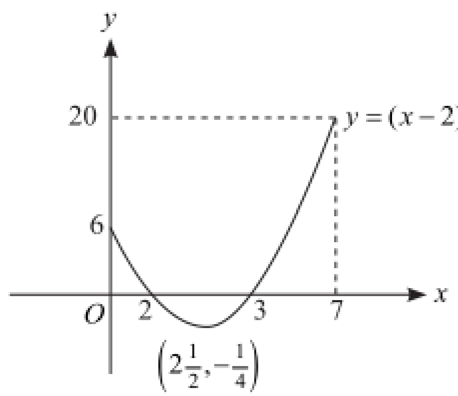
When $x = 0$, $y = 6$ and

when $x = 7$, $y = 20$.

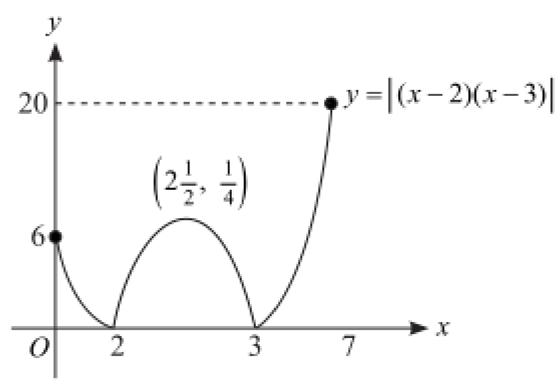
Determine the turning of the graph

Using GC, the turning point is $\left(2\frac{1}{2}, -\frac{1}{4}\right)$.

Sketch $y = (x-2)(x-3)$ for $0 \le x \le 7$.



Then reflect the negative part of the graph in the x-axis to obtain $y = |(x-2)(x-3)|$ for $0 \le x \le 7$.



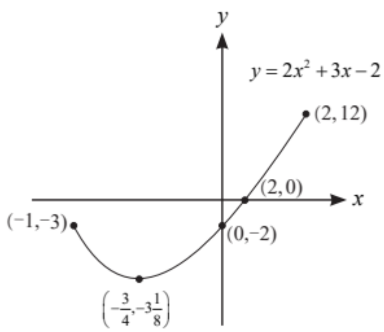
(d) Given $y = 2x^2 + 3x - 2$

Determine axial intercept
When $x = 0$, $y = -2$.
When $y = 0$, $x = 0.5$

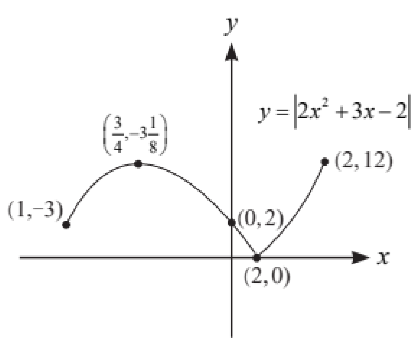
Determine the end-point of the graph
When $x = -1$, $y = -3$ and
when $x = 2$, $y = 12$.

Determine the turning of the graph
Using GC, the turning point is $\left(-\frac{3}{4}, -3\frac{1}{8}\right)$.

Sketch $y = 2x^2 + 3x - 2$ for $-1 \le x \le 2$.



Then reflect the negative part of the graph in the x-axis to obtain $y = \left| 2x^2 + 3x - 2 \right|$ for $-1 \le x \le 2$.



Exercise 4

C Solving Inequalities involving Modulus

21
Solution

(a) $|5x-7|>2x+1$

$5x-7>2x+1$ or $-(5x-7)<2x+1$

$3x>8$ or $7x<6$

$\therefore\ x<\frac{6}{7}$ or $x>\frac{8}{3}$

(b) $|x|<4|x-3|$ ◁ square both sides when modulus on both sides

$x^2<[4(x-3)]^2$

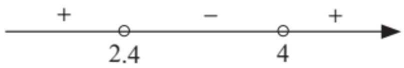
$x^2<16(x^2-6x+9)$

$x^2<16x^2-96x+144$

$15x^2-96x+144>0$

$3x^2-32x+48>0$

$(x-4)(5x-12)>0$



$\therefore\ x>4$ or $x<2.4$

(c) $\left|\frac{7x-1}{3-x}\right|<1$ ◁ cross multiply when the denominator is always positive

$|7x-1|<|3-x|$

$(7x-1)^2<(3-x)^2$

$49x^2-14x+1<9-6x+x^2$

$48x^2-8x-8<0$

$6x^2-x-1<0$

$(2x-1)(3x+1)<0$

$\therefore\ -\frac{1}{3}<x<\frac{1}{2}$

(d) $x^2 - |x| - 5 > 0$ $\quad \triangleleft$ note $x^2 = |x|^2$

$|x|^2 - |x| - 5 > 0$

Let $y = |x|$

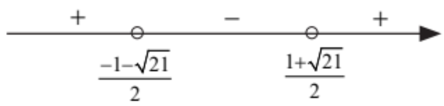
$y^2 - y - 5 > 0$

Let $y^2 - y - 5 = 0$

$$y = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-5)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{21}}{2}$$

So, $\left(x - \dfrac{1+\sqrt{21}}{2}\right)\left(x - \dfrac{1-\sqrt{21}}{2}\right) > 0$



$y > \dfrac{1+\sqrt{21}}{2}$ or $y < \dfrac{1-\sqrt{21}}{2}$

Replace y with $|x|$

$|x| > \dfrac{1+\sqrt{21}}{2}$ or $|x| < 1 - \dfrac{\sqrt{21}}{2}$ (no solution since $|x| > 0$)

$\therefore \; x > \dfrac{1+\sqrt{21}}{2}$ or $x < \dfrac{-1-\sqrt{21}}{2}$

22
Solution

Given $$\frac{x+2}{x-3} \geq \frac{x-4}{(x-3)^2}$$

$$\frac{x+2}{x-3} - \frac{x-4}{(x-3)^2} \geq 0$$

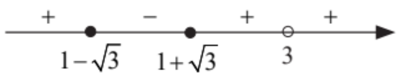
$$\frac{(x+2)(x-3)-x+4}{(x-3)^2} \geq 0$$

$$\frac{x^2-2x-2}{(x-3)^2} \geq 0$$

$$\frac{(x-1)^2-3}{(x-3)^2} \geq 0$$

$$\frac{(x-1+\sqrt{3})(x-1-\sqrt{3})}{(x-3)^2} \geq 0$$

$$\frac{(x-(1-\sqrt{3}))(x-(1+\sqrt{3}))}{(x-3)^2} \geq 0$$



$\therefore\ x \leq 1-\sqrt{3}$ or $1+\sqrt{3} \leq x < 3$ or $x > 3$

Given $$\frac{|x|+2}{|x|-3} \geq \frac{|x|-4}{(|x|-3)^2}$$

Let $y = |x|$

$$\frac{y+2}{y-3} \geq \frac{y-4}{(y-3)^2}$$

Use the result from earlier part

$$y \leq 1-\sqrt{3} \quad \text{or} \quad 1+\sqrt{3} \leq y < 3 \quad \text{or} \quad y > 3$$

$$|x| \leq 1-\sqrt{3} \quad \text{or} \quad 1+\sqrt{3} \leq |x| < 3 \quad \text{or} \quad |x| > 3$$

$|x| \leq 1-\sqrt{3}$ (no solution since $|x| \geq 0$)

$\therefore\ x \leq -1-\sqrt{3},\ x \neq -3$ or $x \geq 1+\sqrt{3},\ x \neq 3$

23
Solution

Given $\dfrac{1+6x}{3-x} < 1+6x$

$\dfrac{1+6x}{3-x} - (1+6x) < 0$

$(1+6x)\left[\dfrac{1-(3-x)}{3-x}\right] < 0$

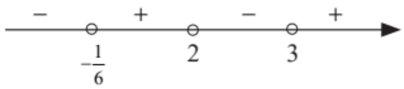
$\dfrac{(1+6x)(x-2)}{3-x} < 0$

$(1+6x)(3-x)(x-2) < 0$

$(1+6x)(x-3)(x-2) > 0$

$-\dfrac{1}{6} < x < 2 \quad \text{or} \quad x > 3$...................................... (1)

$\{x : x \in \mathbb{R},\ -\dfrac{1}{6} < x < 2 \text{ or } x > 3\}$



Replace x in (1) by $|x|$.

$-\dfrac{1}{6} < |x| < 2 \quad \text{or} \quad |x| > 3$

$-\dfrac{1}{6} < |x| \quad \text{and} \quad |x| < 2 \qquad \text{or} \quad |x| > 3$

$x \in \mathbb{R} \quad \text{and} \quad -2 < x < 2 \quad \text{or} \quad x < -3 \quad \text{or} \quad x > 3$

$\therefore \quad -2 < x < 2 \quad \text{or} \quad x < -3 \quad \text{or} \quad x > 3$

24
Solution

(a) $x^2 - x + 1$

$= x^2 - x + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 1$

$= \left[x - \left(\frac{1}{2}\right)\right]^2 + \frac{3}{4}$

$= \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$

Since $\left(x - \frac{1}{2}\right)^2 \geq 0$ for all $x \in \mathbb{R}$, then $\left(x - \frac{1}{2}\right)^2 + \frac{3}{4} > 0$ for all $x \in \mathbb{R}$.

Hence $x^2 - x + 1$ is always positive for all real values of x.

(b)

$$\frac{2x+5}{4+3x-x^2} \leq 1$$

$$\frac{2x+5-(4+3x-x^2)}{4+3x-x^2} \leq 0$$

$$\frac{2x+5-4-3x+x^2}{4+3x-x^2} \leq 0$$

$$\frac{x^2-x+1}{4+3x-x^2} \leq 0$$

$$\frac{x^2-x+1}{(4-x)(x+1)} \leq 0,\ x \neq 4,\ x \neq -1 \ ....\ (1)$$

Since $x^2 - x + 1 > 0$ for all $x \in \mathbb{R}$.

$$\frac{1}{(4-x)(x+1)} < 0$$

$$\frac{1}{(x-4)(x+1)} > 0$$

$$(x-4)(x+1) > 0$$

$\therefore\ x < -1$ or $x > 4$

(c) To solve inequality $\frac{2|x|+5}{4+3|x|-x^2} \leq 1$.

$$\frac{2|x|+5}{4+3|x|-|x|^2} \leq 1$$

Replace x in (1) by $|x|$.

So, $\frac{2|x|+5}{4+3|x|-|x|^2} \leq 1$

Deduce the result in (*). Replace x by $|x|$ in (*).

$|x| < -1$ or $|x| > 4$

$|x| < -1$ (no solution since $|x| \geq 0$ for all $x \in \mathbb{R}$) or $|x| > 4$

$\therefore\ x < -4$ or $x > 4$

25
Solution

$$\frac{x}{2x+1}-\frac{x+3}{1-x}$$

$$=\frac{x}{2x+1}+\frac{x+3}{x-1}$$

$$=\frac{x(x-1)+(x+3)(2x+1)}{(2x+1)(x-1)}$$

$$=\frac{x^2-x+2x^2+7x+3}{(2x+1)(x-1)}$$

$$=\frac{3x^2+6x+3}{(2x+1)(x-1)}$$

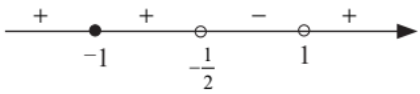
$$=\frac{3(x^2+2x+1)}{(2x+1)(x-1)}$$

$$=\frac{(x+1)^2}{(2x+1)(x-1)}$$

Given $\dfrac{x+3}{1-x}\geq\dfrac{x}{2x+1}$ .... (1)

$\dfrac{x}{2x+1}-\dfrac{x+3}{1-x}\leq 0$ ◁ use the above result

$\dfrac{(x+1)^2}{(2x+1)(x-1)}\leq 0$



$\therefore\ -\dfrac{1}{2}<x<1$ or $x=-1$ .... (*)

To solve the inequality $\dfrac{3-|x|}{1+|x|}\geq\dfrac{|x|}{2|x|-1}$.

Replace x by $-|x|$ in (1).

$$\frac{-|x|+3}{1-(-|x|)}\geq\frac{-|x|}{2(-|x|)+1}$$

$$\frac{3-|x|}{1+|x|)}\geq\frac{-|x|}{-2|x|+1}$$

$$\therefore\quad \frac{3-|x|}{1+|x|}\geq\frac{|x|}{2|x|-1}$$

Deduce the result in (*). Replace x with $(-|x|)$ in (*).

$\therefore\ -\dfrac{1}{2}<-|x|<1$ or $-|x|=-1$

$-1<|x|<\dfrac{1}{2}$ or $|x|=1$

$-\dfrac{1}{2}<x<\dfrac{1}{2}$ or $x=1$ or $x=-1$

$\therefore\ -\dfrac{1}{2}<x<\dfrac{1}{2}$ or $x=1$ or $x=-1$

26
Solution

Given $\dfrac{1}{x-2} \le \dfrac{3}{5x+2}$ .... (1)

$$\frac{1}{x-2} - \frac{3}{5x+2} \le 0$$

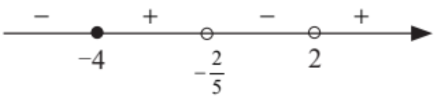
$$\frac{(5x+2)-3(x-2)}{(x-2)(5x+2)} \le 0$$

$$\frac{5x+2-3x+6}{(x-2)(5x+2)} \le 0$$

$$\frac{2x+8}{(x-2)(5x+2)} \le 0$$

$$\frac{x+4}{(x-2)(5x+2)} \le 0$$

$$(x+4)(x-2)(5x+2) \le 0$$



$\therefore\ x \le -4$ or $-\dfrac{2}{5} < x < 2$ .... (*)

(a) To solve the inequality $\dfrac{1}{|x|-1} \le \dfrac{3}{5|x|+7}$.

Replace x with $|x|+1$ in (1):

$$\frac{1}{|x|+1-2} \le \frac{3}{5(|x|+1)+2}$$

$$\therefore \quad \frac{1}{|x|-1} \le \frac{3}{5|x|+7}$$

Deduce the result in (*). Replace x by $|x|+1$ in (*).

$|x|+1 \le -4$ or $-\dfrac{2}{5} < |x|+1 < 2$

$|x| \le -5$ (no solution since $|x| \ge 0$, for all x) or $-\dfrac{7}{5} < |x| < 1$

$-\dfrac{7}{5} < |x|$ (no solution) and $|x| < 1$

$-1 < x < 1$

$\therefore \quad -1 < x < 1$

(b) To solve the inequality $\dfrac{x}{1-2x} \le \dfrac{3x}{5+2x}$.

Replace x with $\dfrac{1}{x}$ in (1):

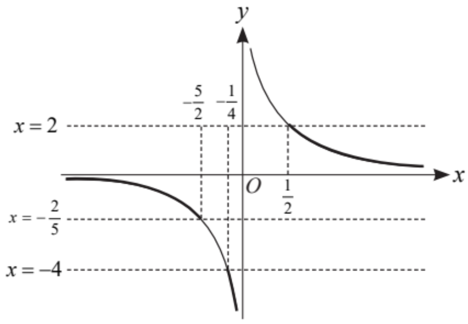
$$\frac{1}{\frac{1}{x}-2} \le \frac{3}{5\left(\frac{1}{x}\right)+2}$$

$$\therefore \quad \frac{x}{1-2x} \le \frac{3x}{5+2x}$$

Deduce the result in (*). Replace x by $\frac{1}{x}$ in (*).

$\frac{1}{x} \le -4$ or $-\frac{2}{5} < \frac{1}{x} < 2$.

Using graphical Method



$\frac{1}{x} \le -4$ or $-\frac{2}{5} < \frac{1}{x} < 2$

From the graph,

$x < -\frac{5}{2}$ or $-\frac{1}{4} \le x \le 0$ or $x > \frac{1}{2}$

27
Solution

Given $\dfrac{y^2-y+6}{y-1}\geq 0$

Since $y^2-y+6=\left\{y-\dfrac{1}{2}\right\}^2+\dfrac{23}{4}>0\ \forall\ y\in\mathbb{R}$.

$\therefore\quad \dfrac{1}{y-1}\geq 0$

$y-1\geq 0$

$\therefore\ y>1$ .... (1)

Given $$\frac{x^2+4x+9}{|x+2|-1}\geq 1$$

$$\frac{x^2+4x+4+5}{|x+2|-1}\geq 1$$

$$\frac{(x+2)^2+5}{|x+2|-1}\geq 1$$

$$\frac{(x+2)^2+5}{|x+2|-1}-1\geq 0$$

$$\frac{(x+2)^2+5-(|x+2|-1)}{|x+2|-1}\geq 0 \qquad \leq (x+2)^2=|x+2|^2 \text{ since } |x|^2=(x)^2$$

$$\frac{|x+2|^2-|x+2|+6}{|x+2|-1}\geq 0$$

Deduce the result in (1). Replace x by $|x+2|$ in (1).

$|x+2|>1$

$x+2<-1$ or $x+2>1$

$x<-3$ or $x>-1$

$\therefore\ x<-3$ or $x>-1$

28
Solution

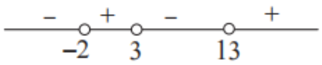
Given $\dfrac{2}{x-3} < \dfrac{3}{x+2}$ .... (1)

$\dfrac{2}{x-3} - \dfrac{3}{x+2} < 0$

$\dfrac{2(x+2)-3(x-3)}{x-3} < 0$

$\dfrac{2x+4-3x+9}{x-3} < 0$

$\dfrac{x-13}{(x-3)(x+2)} > 0$



$(x-13)(x-3)(x+2) > 0,\ x \neq -2, 3$ .... (*)

$\therefore\ x > 13$ or $-2 < x < 3$

To solve the inequality $\dfrac{2}{|x-1|-3} < \dfrac{3}{|x-1|+2}$.

Replace x with $|x-1|$ in (1):

$\therefore\quad \dfrac{2}{|x-1|-3} < \dfrac{3}{|x-1|+2}$

Replace x by $|x-1|$ in (*)

$-2 < |x-1| < 3$ or $|x-1| > 13$

Consider $-2 < |x-1| < 3$.

$-2 < |x-1| < 3$

$-2 < |x-1|$ and $|x-1| < 3$

$x \in \mathbb{R}$ and $-3 < x-1 < 3$

$-2 < x < 4$

Consider $|x-1| > 13$.

$|x-1| > 13$

$x-1 < -13$ or $x-1 > 13$

$x < -12$ or $x > 14$

Taking both results above,

$\therefore\ -2 < x < 4$ or $x < -12$ or $x > 14$

29
Solution

$$\frac{12}{x+1} \le x$$

$$\frac{12}{x+1} - x \le 0$$

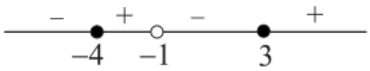
$$\frac{12-(x)(x+1)}{(x+1)} \le 0$$

$$\frac{12-(x^2+x)}{x+1} \le 0$$

$$\frac{12-x^2-x}{x+1} \le 0$$

$$\frac{x^2+x-12}{x+1} \ge 0$$

$(x+4)(x-3)(x+1) \ge 0$



$\therefore \quad -4 \le x < -1$ or $x \ge 3$

Given $\dfrac{12}{|x|+1} \le x$

Note that the inequality cannot hold for all $x < 0$.

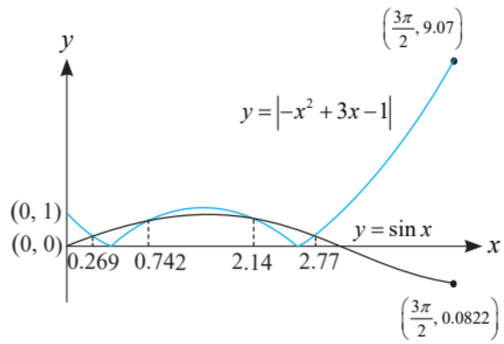
$\dfrac{12}{|x|+1} \le x$ implies $\dfrac{12}{x+1} \le x$ as $x \ge 0$.

$\therefore \quad x \ge 3$

Exercise 4

E Solving Inequalities using graphical Method

30
Solution



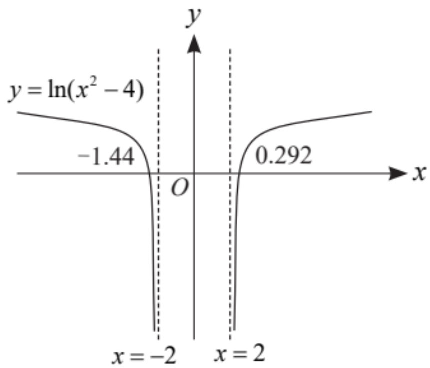
Using GC, points of intersection are at $x = 0.269$, $x = 0.742$, $x = 2.14$ and $x = 2.77$

From the graph, the required inequalities are:

$0 \le x < 0.269$ or $0.742 < x < 2.14$ or $2.77 < x \le \dfrac{3\pi}{2}$.

31
Solution

(a) The graph of $y = \ln(x^2 - 4)$.

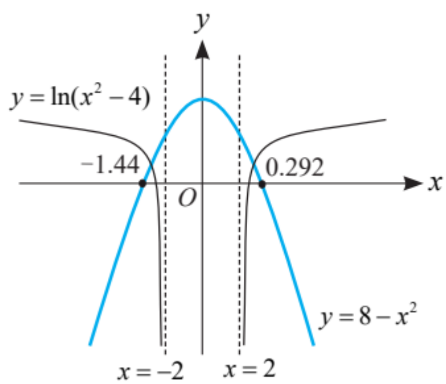


Rewrite $\ln(x^2 - 4) + x^2 - 8 \le 0$

as $\ln(x^2 - 4) \le 8 - x^2$

$\therefore$ the suitable additional curve is $y = 8 - x^2$.

Add $y = 8 - x^2$ on the same diagram.

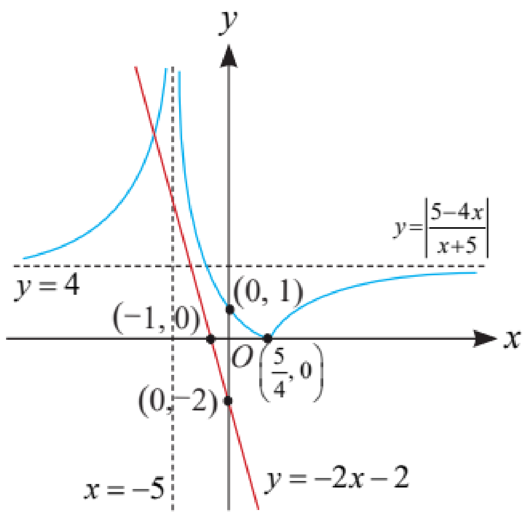


Using GC, the points of intersection between the 2 graph are at $x = 0.292$ and $x = -1.44$.

From the graphs, the required inequalites are :

$-2.63 \le x < -2$ or $2 < x \le 2.63$ (3 s.f.)

32
Solution



(b) Let $-\frac{5-4x}{x+5}=-2x-2$

$$-5+4x=(-2x-2)(x+5)$$

$$-5+4x=-2x^2-12x-10$$

$$2x^2+16x+5=0$$

$$x=\frac{-16\pm\sqrt{16^2-4(2)(5)}}{2(2)}$$

$$x=\frac{-16\pm\sqrt{216}}{4}$$

$$x=-4\pm\frac{3}{2}\sqrt{6}$$

From the graph, the point of intersection occurs when $x<-5$. Hence $x=-4-\frac{3}{2}\sqrt{6}$.

From the graph, the required inequalities are :

$$-4-\frac{3}{2}\sqrt{6}<x<-5 \text{ or } x>-5 \quad \text{.....................} \ (*)$$

(c) Given $\left|\frac{10-4x}{x+10}\right| > -x-2$

$$\left|\frac{2(5-2x)}{2(0.5x+5)}\right| > -x-2$$

$$\left|\frac{5-4\left(\frac{x}{2}\right)}{\left(\frac{x}{2}\right)+5}\right| > -2\left(\frac{x}{2}\right)-2$$

Deduce the result in **(b)**. Replace x with $(\frac{x}{2})$ in (*).

Hence

$$-4-\frac{3}{2}\sqrt{6} < \frac{x}{2} < -5 \quad \text{or} \quad \frac{x}{2} > -5$$

$\therefore \quad -8-3\sqrt{6} < x < -10 \quad \text{or} \quad x > -10$

33
Solution

(a) Given $y = \dfrac{2x^2+3}{x-1}$ ◁ by long division

$$= 2x + 2 + \frac{5}{x-1}$$

Equations of asymptotes

$x = 1$is a vertical asymptote.

$y = 2x + 2$ is an oblique asymptote.

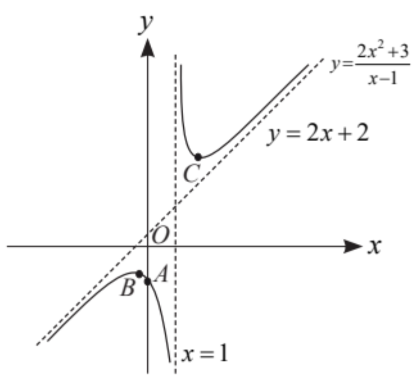
Axial intercept : When $x = 0$, $y = 2$. $\therefore$ y-intercept $A(0, -3)$

No intersection with x-axis

Use G.C. to find turning point.

Minimum point $(2.58,\ 10.3)$ and Maximum point $(-0.581, -2.32)$

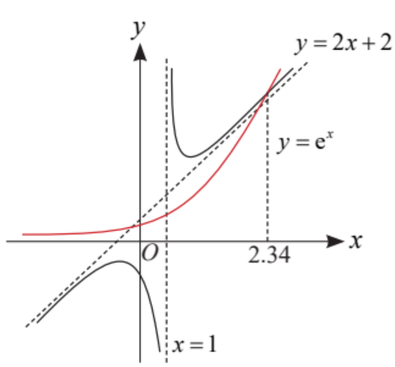
The graph of $y = \dfrac{2x^2+3}{x-1}$.



(b) $$2x + 2 \le e^x - \frac{5}{x-1}$$

$$2x + 2 + \frac{5}{x-1} \le e^x$$

Add the graph $y = e^x$ on the same diagram.



From the graph,
$x < 1$ or $x \geq 2.34$ .... (1)

(c) Deduce the result in **(b)**. Replace x with $x+1$ in ($*$).

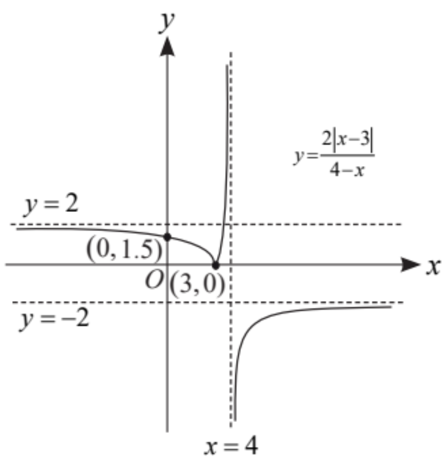
$x+1 < 1$ or $x+1 \geq 2.34$

$x < 0$ or $x \geq 1.34$

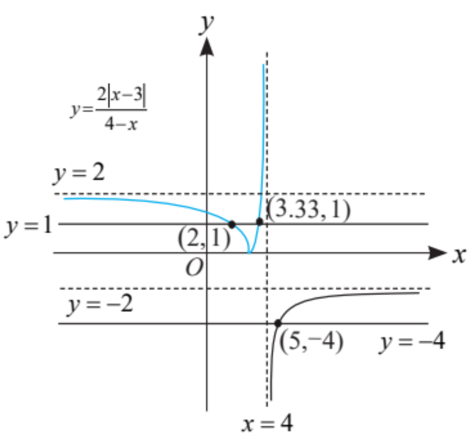
$\therefore$ $x < 0$ or $x \geq 1.34$

34
Solution

(a) The graph of $y = \frac{2|x-3|}{4-x}$.



(b) Added in the line $y = 1$ and $y = -4$.



From the graph, the required inequalities are

$$2 \le x \le 3.33 \quad \text{or} \quad x > 5.$$

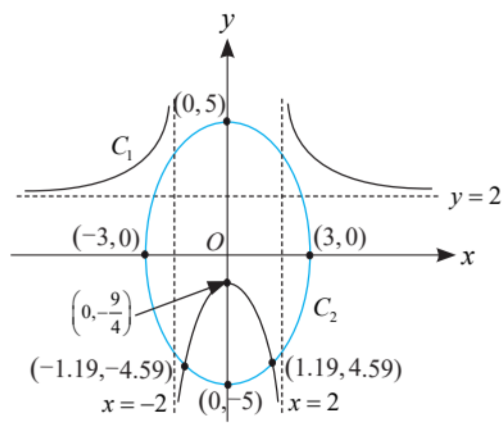
35
Solution

(a) $y = \dfrac{2x^2 + 9}{x^2 - 4}$

$= 2 + \dfrac{17}{(x+2)(x-2)}$

Equations of the asymptotes of the curve C_1 are $x = -2,\ x = 2,\ y = 2$.

(b)



(c) Using GC, the points of intersections of C_1 and C_2 are $(-1.19, -4.59)$ and $(1.19, 4.59)$.

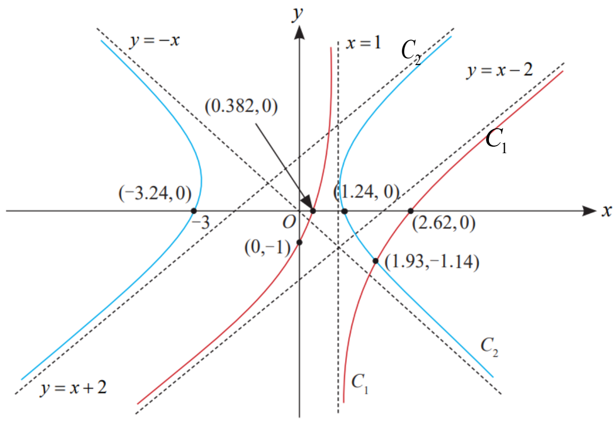
The x-coordinates are -1.19 and 1.19.

(d) From the graph, the required inequalities are

$-3 \le x < -2$ or $-1.19 \le x \le 1.19$ or $2 < x \le 3$

36
Solution

(a)



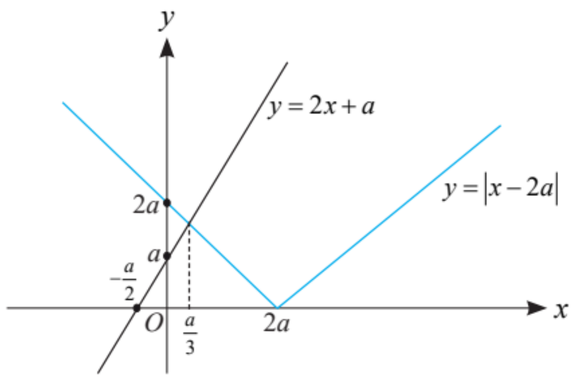
(b) Using GC, the points of intersection of C_1 and C_2 is $(1.93, -1.14)$.

(c) From sketch, using only the bottom part of C_2,
From the graph, the required inequalities are

$$x \le -3 \text{ or } 1 < x \le 1.93$$

37
Solution

Sketch the graphs of $y = |x - 2a|$ and $y = 2x + a$.



To find intersection point, $2x + a = -(x - 2a)$

$$x = \frac{a}{3}$$

From the graph, for $|x - 2a| < 2x + a$, $x > \frac{a}{3}$.

The required inequality is $x > \frac{a}{3}$.

Replace x by $-x$ and let $a = 2$ in $|x - 2a| < 2x + a$.

$\therefore \quad |(-x) - 2(2)| < 2(-x) + 2$

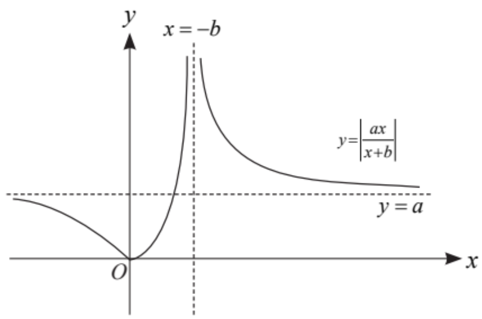
$$|x + 4| < 2 - 2x$$

Thus $\quad -x > \frac{2}{3} \quad$ ◁ (deduced from the previous result)

$\therefore \ x < -\frac{2}{3}$

38
Solution

The graph of $y=\left|\dfrac{ax}{x+b}\right|$, where $b<0$ and $0<a<1$.



Given $\quad |ax| \geq |x+b|$

$$\left|\frac{ax}{x+b}\right| \geq 1$$

Consider $\left|\dfrac{ax}{x+b}\right|=1$.

$\therefore \quad -\dfrac{ax}{x+b}=1$ and $\dfrac{ax}{x+b}=1$

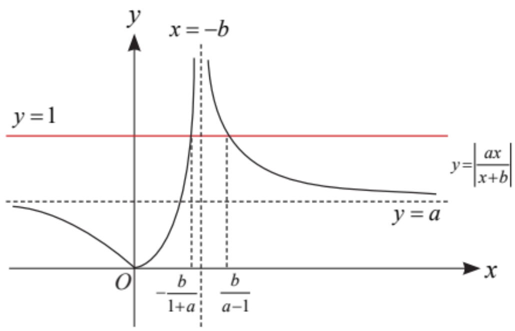
Find the intersection points for $-\dfrac{ax}{x+b}=1$.

$$\begin{aligned} -\frac{ax}{x+b} &= 1 \\ -ax &= x+b \\ -b &= x(1+a) \\ x &= \frac{-b}{1+a} \end{aligned}$$

Find the intersection points for $\dfrac{ax}{x+b}=1$.

$$\begin{aligned} \frac{ax}{x+b} &= 1 \\ ax &= x+b \\ x(a-b) &= b \\ x &= \frac{b}{a-1} \end{aligned}$$

Add the line $y = 1$ on the diagram.



From the graph, for $\left|\dfrac{ax}{x+b}\right| \geq 1$

$\therefore \quad -\dfrac{b}{1+a} \leq x < -b$ or $-b < x \leq \dfrac{b}{a-1}$

Combining gives $-\dfrac{b}{1+a} \leq x \leq \dfrac{b}{a-1}$

$\therefore \quad -\dfrac{b}{1+a} \leq x \leq \dfrac{b}{a-1}$

Learning point:

$-b$ must include in the solution as the question is $|ax| \geq |x+b|$ not $\left|\dfrac{ax}{x+b}\right| \geq 1$.

Common mistake :

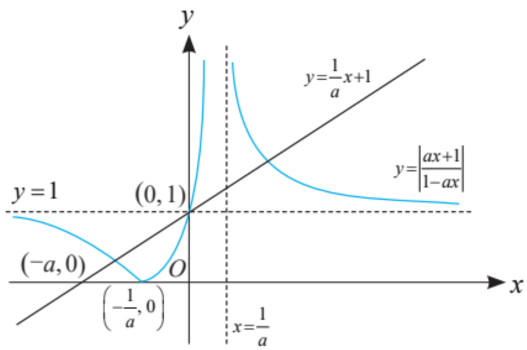
$$\frac{ax}{x+b} \geq 1$$

$$ax \geq x + b$$

Do not cross multiply because $x + b$ may be positive or negative.

39
Solution

(a) The graphs of $y = \left|\dfrac{ax+1}{1-ax}\right|$ and $y = \dfrac{1}{a}x + 1$



(b) The reflected part of $y = \left|\dfrac{ax+1}{1-ax}\right|$ is $-\dfrac{ax+1}{1-ax}$.

Solve $-\dfrac{ax+1}{1-ax} = \dfrac{1}{a}x + 1$ to find the points of intersections.

$$-\frac{ax+1}{1-ax} = \frac{1}{a}x + 1$$

$$-ax - 1 = \left(\frac{1}{a}x + 1\right)(1 - ax)$$

$$-ax - 1 = \frac{1}{a}x + 1 - x^2 - ax$$

$$x^2 - \frac{1}{a}x - 2 = 0$$

$$ax^2 - x - 2a = 0$$

$$x = \frac{1 \pm \sqrt{1 + 8a^2}}{2a}$$

Given $\left|\dfrac{ax+1}{1-ax}\right| > \dfrac{1}{ax} + 1$

From the graph, $x < \dfrac{1 - \sqrt{1+8a^2}}{2a}$ or $0 < x < \dfrac{1}{a}$ or $\dfrac{1}{a} < x < \dfrac{1 + \sqrt{1+8a^2}}{2a}$

Alternative Method :

$$\left(x-\frac{1}{2a}\right)^2=2+\frac{1}{4a^2}=\frac{8a^2+1}{4a^2}$$

$$x=\frac{1}{2a}\pm\frac{\sqrt{8a^2+1}}{2a}$$

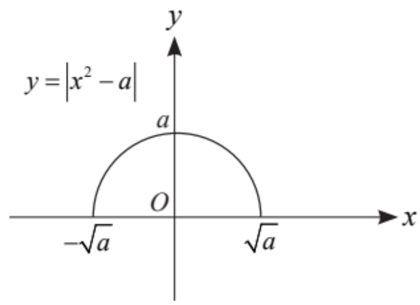
$$=\frac{1\pm\sqrt{8a^2+1}}{2a}$$

Given $\left|\frac{ax+1}{1-ax}\right|>\frac{1}{ax}+1$

From the graph, $x<\frac{1-\sqrt{1+8a^2}}{2a}$ or $0<x<\frac{1}{a}$ or $\frac{1}{a}<x<\frac{1+\sqrt{1+8a^2}}{2a}$.

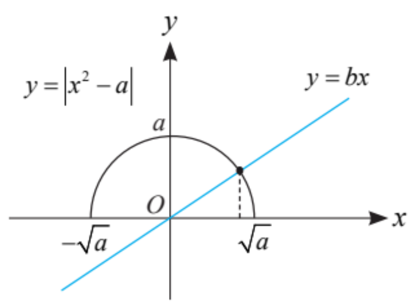
40
Solution

The graph of $y = \left| x^2 - a \right|$, where $-\sqrt{a} \le x \le \sqrt{a}$



For $\left| x^2 - a \right| \le bx$.

Add the graph of $y = bx$.



Consider $\left| x^2 - a \right| = bx$.

From the graph, $y \ge 0$

Let $\quad -(x^2 - a) = bx$

$$x^2 + bx - a = 0$$

$$x = \frac{-b \pm \sqrt{b^2 + 4a}}{2}$$

Since $x > 0$, $x = \dfrac{-b + \sqrt{b^2 + 4a}}{2}$.

The two curves intersect at $x = \dfrac{-b + \sqrt{b^2 + 4a}}{2}$.

From the graph, the required inequality is $\dfrac{-b + \sqrt{b^2 + 4a}}{2} \le x \le \sqrt{a}$.

Exercise 4

F Solving System Linear Equations

41
Solution

(a) Let $y = ax^2 + bx + c$ .... (1)

The graph passes through the points $(1, 8)$, $(-1, 0)$ and $(-2, 11)$.

Substitute $(1, 8)$ into (1):

$$8 = a + b + c$$

Substitute $(-1, 0)$ into (1):

$$0 = a - b + c$$

Substitute $(-2, 11)$ into (1):

$$11 = 4a - 2b + c$$

$\therefore$ the system of linear equations involving a, b and c as follows:

$$8 = a + b + c \quad \text{.........................} \ (2)$$
$$0 = a - b + c \quad \text{.........................} \ (3)$$
$$11 = 4a - 2b + c \quad \text{.....................} \ (4)$$

(b) Using GC to solve (2), (3) and (4).

$\therefore$ $a = 5$, $b = 4$, $c = -1$

42
Solution

Let $\mathrm{f}(x) = x^3 + ax^2 + bx + c$

When $\mathrm{f}(x)$ is divided by $(x-1)$, the remainder is 8.

i.e. $\mathrm{f}(1) = 8$

$1 + a + b + c = 8$

$a + b + c = 7$ .... (1)

When $\mathrm{f}(x)$ is divided by $(x-2)$, the remainder is 12.

$\mathrm{f}(2) = 12$

$8 + 4a + 2b + c = 12$

$4a + 2b + c = 4$ .... (2)

When $\mathrm{f}(x)$ is divided by $(x-3)$, the remainder is 25.

$\mathrm{f}(3) = 25$

$27 + 9a + 3b + c = 25$

$9a + 3b + c = -2$ .... (3)

Using GC to solve (1), (2) and (3).

$\therefore\ a = -1.5,\ b = 1.5,\ c = 7$

43
Solution

Let $u_n = an^2 + bn + c$

When $u_1 = 10,\quad a+b+c = 10$ .... (1)

When $u_2 = 6, 4a+2b+c = 6$ .... (2)

When $u_3 = 5\ \ 9a+3b+c = 5$ .... (3)

Using GC to solve (1), (2) and (3).

$\therefore\ a = \frac{3}{2}, b = -\frac{17}{2}, c = 17$

Hence, $u_n = \frac{3}{2}n^2 - \frac{17}{2}n + 17$

Given u_n is greater than 100.

i.e. $\frac{3}{2}n^2 - \frac{17}{2}n + 17 > 100$

Using GC, $n < -5.12$ or $n > 10.79$

Hence $\{n \in \mathbb{Z}^+, n \geq 11\}$.

44
Solution

Let the equation of the curve as $y = ax^3 + bx^2 + cx + d$

The curve passes through $\left(-2, \ \frac{34}{3}\right)$

i.e. $a(-2)^3 + b(-2)^2 + c(-2) + d = \frac{34}{3}$

$$-8a + 4b - 2c + d = \frac{34}{3} \text{ .... (1)}$$

The curve passes through $\left(3, -\frac{19}{2}\right)$.

i.e. $\quad a(3)^3 + b(3)^2 + c(3) + d = -\frac{19}{2}$

$$27a + 9b + 3c + d = -\frac{19}{2} \text{ .... (2)}$$

Differentiate y with respect to x.

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3ax^2 + 2bx + c.$$

The curve has a maximum point at $\left(-2, \ \frac{34}{3}\right)$.

Substitute $x = -2$ and $y = \frac{34}{3}$ into $\frac{\mathrm{d}y}{\mathrm{d}x}$.

$3a(-2)^2 + 2b(-2) + c = 0$

$$12a - 4b + c = 0 \text{ ....(3)}$$

The curve has a minimum point $\left(3, -\frac{19}{2}\right)$.

Substitute $x = 3$ and $y = -\frac{19}{2}$ into $\frac{\mathrm{d}y}{\mathrm{d}x}$.

$3a(3)^2 + 2b(3) + c = 0$

$$27a + 6b + c = 0 \text{ ....(4)}$$

Using GC to solve (1), (2), (3) and (4).

Solving $a = \frac{1}{3}, \ b = -\frac{1}{2}, \ c = -6, \ d = 4$

$\therefore$ the equation of the curve is $y = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 6x + 4$

Exercise 4

G Words problems involving System Linear Equations

45
Solution

Let $\$P$, $\$H$, $\$T$ be the cost of 1 kg of picnic gammon, honey-baked gammon and turkey gammon respectively.

$1.5P+0.6H+2.25T=4.89$ .... (1)

$0.5P+2H+1.65T=4.87$ .... (2)

$2.45P+1.45H+0.8T=4.79$ .... (3)

Using GC to solve (1), (2) and (3):

$$P=0.85,\ H=1.15,\ T=1.3$$

Total amount that Mrs Teo paid

$=1.3(0.85)+0.9(1.15)+2.1(1.3)$

$=\$4.87$

46
Solution

Let x, y, z be the cost for 1 KWh of electricity consumed, 1 KWh of gas consumed and 1 m^3 of water use respectively.

$1521x+103y+35.6z=459.81$ .... (1)

$1806x+68y+41.1z=533.16$ .... (2)

$1089x+97y+33.0z=343.11$ .... (3)

Using GC to solve (1), (2) and (3):

$$x=0.26,\ y=0.21,\ z=1.2$$

Let k be the amount of gas usage on Denise's bill.

$$1616(0.26)+k(0.21)+38.2(1.2)=481.83$$

$$k=75.381$$

$$=75.4 \text{ (to 3 s.f.)}$$

Denise's household used 75.4 k Wh of gas in that month.

47
Solution

Let $\$a,\ \$b,\ \$c$ be the amount of money Mr Li invested in Bank $A,\ B$ and C respectively.

$$a+b+c=140000 \text{ .... } (1)$$

$$b=0.5c$$

$$b-0.5c=0 \text{ .... } (2)$$

Total amount of money Mr Li had in Bank $A,\ B$ and C at the end of first year

$$a+x+1.015b+c=140800 \text{ .... } (3)$$

Total amount of money Mr Li had in Bank $A,\ B$ and C at the end of second year

$$a+2x+(1.015)^2 b+c=141609 \text{ .... } (4)$$

Using GC to solve (1), (2) (3) and (4):

$$a=20000,\ b=40000,\ c=80000 \text{ and } x=200$$

Total amount of money Mr Li has at the end of 5 years

$=(\$20000+5(\$200))+((1.015)^5\times\$40000)+(1.05)(\$80000)$

$=\$148091.36$

The total amount of money Mr Li has in the three banks at the end of five years is \$148091.36.

Learning point:

In this context (money), the "non-exact" amount should be rounded off to 2 decimal places unless otherwise stated.

48
Solution

Let x m, y m and z m be the length of each section of the track.

$$x+y+z=2400 \text{ .... } (1)$$

Time taken for John took to complete the track:

$$\frac{x}{3.20}+\frac{y}{2.50}+\frac{z}{1.25}=25\times60+58$$

i.e. $$\frac{x}{3.20}+\frac{y}{2.50}+\frac{z}{1.25}=1558 \text{ .... } (2)$$

Time taken for Mary took to complete the track:

$$\frac{x}{5}+\frac{y}{4}+\frac{z}{2.50}=13\times60+36$$

i.e. $$\frac{x}{5}+\frac{y}{4}+\frac{z}{2.50}=816 \text{ .... } (3)$$

Using GC to solve (1), (2) and (3):

$$x=480,\ y=320 \text{ and } z=1600.$$

$\therefore$ the length of section $A,\ B$ and C are 480 m, 320 m and 1600 m respectively.

49
Solution

(a) At junction A: $b+c=a+d$

$$-a+b+c-d=0 \text{ .... (1)}$$

At junction B: $a+b+c=48$

$$a+b+c=48 \text{ .... (2)}$$

At junction C: $a+c=2b$

$$a-2b+c=0 \text{ .... (3)}$$

At junction D: $d=b+a+a$

$$2a+b-d=0 \text{ .... (4)}$$

Using GC to solve (1), (2), (3) and (4):

$$a=8,\ b=16,\ c=24 \text{ and } d=32.$$

(b) Total revenue collected

$=\$0.50(2c+b)$

$=\$0.50(48+16)$

$=\$32$

Total revenue collected on that day is \$32.

50
Solution

Let x, y, z be the number of curry puffs, spring rolls and chicken wings that Sam bought respectively.

$$x+y+z=20 \text{ .... (1)}$$

$$3x+4y+5z=76 \text{ .... (2)}$$

Using GC to solve (1) and (2):

$$x=4+z$$

$$y=16-2z$$

$$z=z$$

Given that there are at least 5 portions,

$$y \geq 5$$

$$16-2z \geq 5$$

$$z \leq 5.5$$

$\therefore\ z=5.$

Substituting $z=5$ into $x=4+z$ and $y=16-2z$.

$\therefore\ x=9$ and $y=6.$

He bought 9 curry puffs, 6 spring rolls and 5 chicken wings.

51
Solution

Let the number polo, casual, and formal shirts be x, y and z respectively.

$3x+4y+5z=98$ .... (1)

$x+y+z=20$ .... (2)

Using GC to solve (1) and (2):

$$x=-18+z$$
$$y=38-2z$$
$$z=z$$

Since x, y and z are all non-negative integers,

For $x\geq 0$, i.e. $-18+z\geq 0$

$z\geq 18$

For $y\geq 0$, i.e. $38-2z\geq 0$

$z\leq 19$

$\therefore\ z=18$ or 19

Hence the possible combinations are:

$z=18, x=0, y=2$

$z=19, x=1, y=0$

For $x=0, y=2, z=18$, gain $=\$(0\times 80+2\times 130+18\times 150)=\$2960$

For $x=1, y=0, z=19$, gain $=\$(1\times 80+0\times 130+19\times 150)=\$2930$

The company should produce 2 casual shirts and 18 formal shirts.

Exercise 4

H Mixed Exercise

52
Solution

(a) Given $y = \dfrac{2(x^2-8)}{x-3}$

$$y(x-3) = 2x^2 - 16$$
$$xy - 3y = 2x^2 - 16$$
$$2x^2 - xy + 3y - 16 = 0$$

For the values of y can take, it implies that there are real roots,
i.e. Discriminant ≥ 0

$$y^2 - 4(2)(3y-16) \geq 0$$
$$y^2 - 24y + 128 \geq 0$$
$$(y-16)(y-8) \geq 0$$

$\{y \in \mathbb{R} : y \leq 8 \text{ or } y \geq 16\}$

(b) $y = \dfrac{2(x^2-8)}{x-3}$

$$= 2x + 6 + \frac{2}{x-3}$$

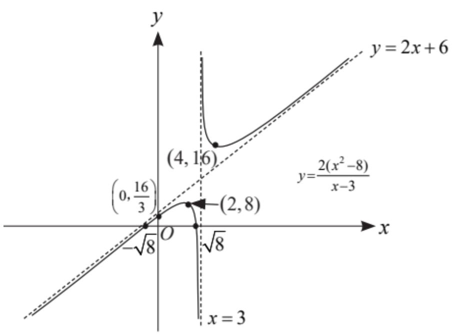
Equations of asymptotes
$x = 3$ is a vertical asymptote.
$y = 2x + 6$ is an oblique asymptote.

Axial intercept : When $x = 0$, $y = \dfrac{16}{3}$

When $y = 0$, $x = \sqrt{8}$ or $-\sqrt{8}$

Use G.C. to find turning point.
Minimum point $(4, 16)$ and maximum point $(2, 8)$

The graph of $y = \dfrac{2(x^2-8)}{x-3}$.

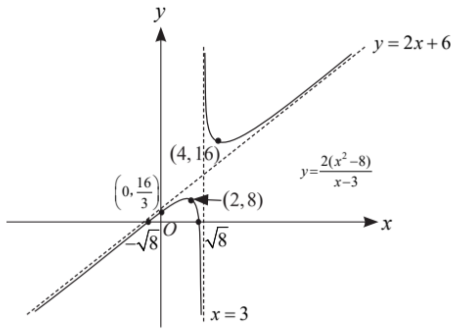


(c) $2x^2 - 16 = k(x-3)^2 + 12(x-3)$

$$\frac{2x^2-16}{(x-3)} = k(x-3)+12$$

$$y = k(x-3)+12$$

$$y = kx - 3k + 12$$



Refer to the graph. For the equation $\dfrac{2x^2-16}{(x-3)} = k(x-3)+12$ has no solution, the gradient of the line $y = kx - 3k + 12$ must be equal or less than 2.

$\therefore\ k \le 2.$

Alternative Method

$2x^2 - 16 = k(x-3)^2 + 12(x-3)$

$2x^2 - 16 = kx^2 - 6kx + 9k + 12x - 36$

$0 = (k-2)x^2 + (12-6k)x + (9k-20)$ .... (1)

For no solution, the discriminant of (1) < 0.

i.e. $(12-6k)^2 - 4(k-2)(9k-20) < 0$

$36k^2 - 144k + 144 - 36k^2 + 152k - 160 < 0$

$8k - 16 < 0$

$k < 2$

Also, if $k = 2$, $(k-2)x^2 + (12-6k)x + (9k-20) = -2 \ne 0$

$\therefore$ the equation has no solution when $k = 2$.

$\therefore\ k \le 2$

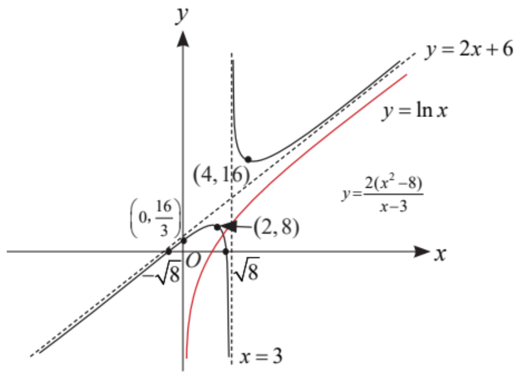
(d) Sketch $y = \ln x$

By G.C., the points of intersection are 2.81 and 3.

From the diagram, for $\dfrac{2(x^2-8)}{x-3} < \ln x$

the inequality is $2.81 < x < 3$ (correct to 3 s.f.)

$\therefore\ 2.81 < x < 3$



Replace x with $|x|$ in $2.81 < x < 3$.

$$2.81 < |x| < 3$$

$$2.81 < |x| \quad \text{and} \quad |x| < 3$$

$$x > 2.81 \text{ or } x < -2.81 \quad \text{and} \quad -3 < x < 3$$

$\therefore \; -3 < x < -2.81$ or $2.81 < x < 3$ (correct to 3 s.f.)

(e) $y = 2x + 6 + \dfrac{2}{x-3}$

$\downarrow$ replace x by $x+3$

$y = 2x + 12 + \dfrac{2}{x}$

$\downarrow$ replace y by $y+12$

$y = 2x + \dfrac{2}{x}$

Description of sequence of transformations

Translate in the negative x-direction by 3 units, followed by

Translate in the negative y-direction by 12 units

53
Solution

(a) Since $\mathrm{R_h} = (0,\ \infty),\ \mathrm{D_g} = \mathbb{R}$

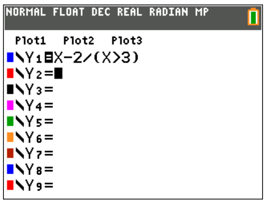
$\mathrm{R_h} \subseteq \mathrm{D_g}$, gh exists.

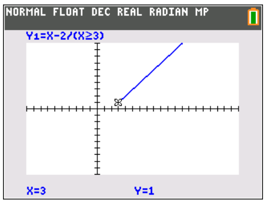
$$\mathrm{gh}(x) = \mathrm{g}\left[\ln(x-2)\right]$$
$$= \mathrm{e}^{\ln(x-2)}$$

$$\mathrm{gh} : x \mapsto x-2,\ x \in \mathbb{R},\ x > 3$$

Use GC to graph $\mathrm{gh}(x) = x-2$, where $x > 3$

From the graph, $\mathrm{R_{gh}} = (1,\ \infty)$





(b) Given $\quad \mathrm{f}(x+a) > \mathrm{gh}(x+a)$

$$(x+a-a)(x+a-2) > x+a-2$$
$$(x)(x+a-2)-(x+a-2) > 0$$
$$(x+a-2)(x-1) > 0$$
$$\therefore\ x < 1 \text{ or } x > 2-a$$

Note that $\mathrm{D_{gh}} = (3,\ \infty)$.

Hence range of values of x is $x > 3$.

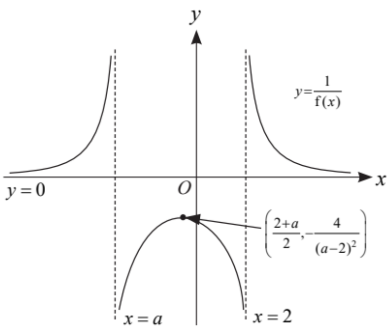
Replacing x by $x+a$

$\therefore\quad x+a > 3$

$\qquad x > 3-a$

$\therefore$ the range of values of x for which $\mathrm{f}(x+a) > \mathrm{gh}(x+a)$ is $(3-a,\ \infty)$

(c) The graph of $y = \dfrac{1}{\mathrm{f}(x)}$



54
Solution

(a) Given $y = \dfrac{x^2+x-2}{x+k}$

$$\frac{dy}{dx} = \frac{x^2+2kx+(k+2)}{(x+k)^2}$$

At stationary points, $\dfrac{dy}{dx} = 0$.

i.e. $\dfrac{x^2+2kx+(k+2)}{(x+k)^2} = 0$

$\therefore \quad x^2+2kx+(k+2) = 0$

For the curve to have 2 stationary points, $x^2+2kx+(k+2)=0$ need to have 2 distinct real roots

i.e. Discriminant ≥ 0

So, $(2k)^2 - 4(1)(k+2) > 0$

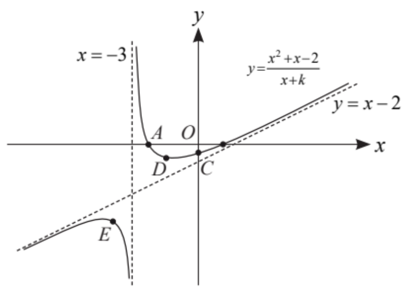
$4k^2 - 4k - 8 > 0$

$k^2 - k - 2 > 0$

$(k+1)(k-2) > 0$

$\therefore$ range of values of k is $k < -1$ or $k > 2$.

(b) Sketch of $y = \dfrac{x^2+x-2}{x+3} = x - 2 + \dfrac{4}{x+3}$



The x-intercepts are at points: $A(-2,\ 0)$; $B(1,\ 0)$

The y-intercepts is at point: $C\left(0, -\dfrac{2}{3}\right)$

The minimum pt is at $D(-1,-1)$ and the maximum point is at $E(-5,-9)$.

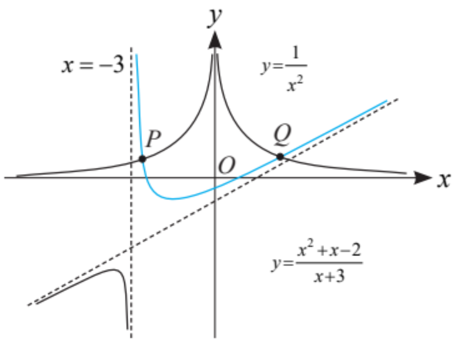
(c) Given $x^4 + x^3 - 2x^2 - x - 3 \le 0$

$$x^4 + x^3 - 2x^2 \le x + 3$$

$$x^2(x^2 + x - 2) \le x + 3$$

$$\frac{x^2 + x - 2}{x + 3} \le \frac{1}{x^2}$$

The additional graph is $y = \frac{1}{x^2}$ for $x \in \mathbb{R}\backslash\{0\}$



Using GC, the x coordinates of intersecting points P and Q are -2.07 and 1.54 respectively.

Refer to the above diagram.

For $y = \frac{x^2 + x - 2}{x + 3} \le \frac{1}{x^2}$ and $x > -3$.

The inequalites are $-2.07 \le x < 0$ or $0 < x \le 1.54$.

However, when $x = 0$, it satisfies $x^4 + x^3 - 2x^2 - x - 3 \le 0$.

Hence, the inequality is $-2.07 \le x \le 1.54$.

55
Solution

(a) $y = -4x|x+a|$

$\downarrow$ Reflection in the y-axis (replace with $-x$)

$= 4x|-x+a|$

$= 4x|x-a|$

$\downarrow$ Scaling by factor $\frac{1}{2}$ parallel to the y-axis $\left(\text{replace } y \text{ with } 2y\right)$

$2y = 4x|x-a|$

$y = 2x|x-a|$

Description of sequence of transformations

1. Reflect $y = \mathrm{f}(x)$ in the y-axis.

2. Scale the resulting curve by a factor $\frac{1}{2}$ parallel to the y-axis.

Alterative solution:

$y = -4x|x+a|$

$\downarrow$ Scaling by factor $\frac{1}{2}$ parallel to the y-axis $\left(\text{replace } y \text{ with } 2y\right)$

$2y = -4x|x+a|$

$y = -2x|x+a|$

$\downarrow$ Reflection in the y-axis (replace with $-x$)

$y = 4x|(-x)+a|$

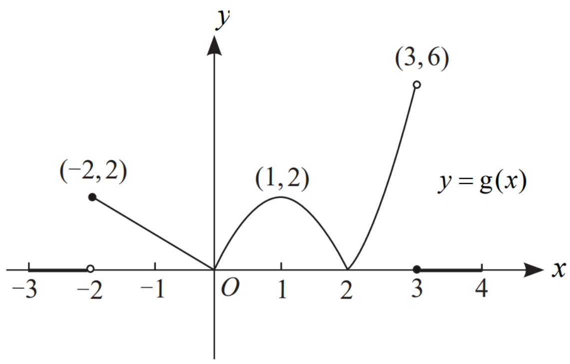
$y = 2x|x-a|$

Description of sequence of transformations

1. Scale the resulting curve by a factor $\frac{1}{2}$ parallel to the y-axis

2. .Reflect $y = \mathrm{f}(x)$ in the y-axis.

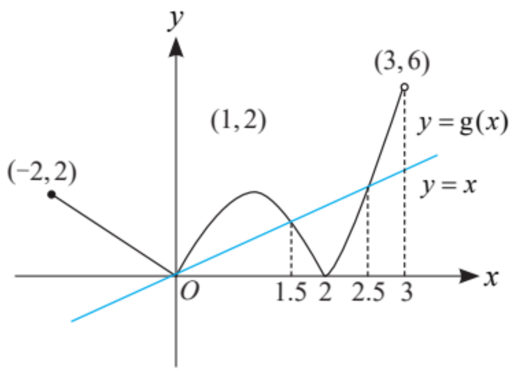
(b) The graph $y = g(x)$ for $-3 \le x \le 4$.



(c) Given

$$\begin{aligned} g(x) &> x \\ 2x|x-2| &> x \quad \triangleleft \text{ square both sides} \\ 4x^2(x-2)^2 &> x^2 \\ 4(x-2)^2 &> 1 \\ (x-2)^2 - \frac{1}{4} &> 0 \\ \left(x-\frac{3}{2}\right)\left(x-\frac{5}{2}\right) &> 0 \\ x < \frac{3}{2} \text{ or } x &> \frac{5}{2} \end{aligned}$$

Add the line $y = x$ on the same diagram.



From the graph, for $g(x) > x$, where $0 \le x < 3$

The set of values of x is $\left\{x : x \in \mathbb{R},\ 0 < x < \frac{3}{2} \text{ or } \frac{5}{2} < x < 3\right\}$

56
Solution

(a) Let $f(x) = ax^3 + bx^2 + cx + d$ .... (1)

The curve $y = f(x)$ passes through (2, 0), (5, 0) and (4, −4).

Substitute the point (2, 0) into (1).

$$a(2)^3 + b(2)^2 + c(2) + d = 0$$
$$8a + 4b + 2c + d = 0 \text{ .... (2)}$$

Substitute the point (5, 0) into (1).

$$a(5)^3 + b(5)^2 + c(5) + d = 0$$
$$125a + 25b + 5c + d = 0 \text{ .... (3)}$$

Substitute the point (4, −4) into (1).

$$a(4)^3 + b(4)^2 + c(4) + d = -4$$
$$64a + 16b + 4c + d = -4 \text{ .... (4)}$$

Differentiate (1) with respect to x.

$f'(x) = 3ax^2 + 2bx + c$

At minimum point, $f'(x) = 0$

Substitute the point (4, −4) into $f'(x) = 0$

$$f'(4) = 3a(4)^2 + 2b(4) + c = 0$$
$$48a + 8b + c = 0 \text{ .... (5)}$$

Use GC to solve (2), (3), (4) and (5).

$\therefore\ a = 1,\ b = -9,\ c = 24,\ d = -20$

Hence, $f(x) = x^3 - 9x^2 + 24x - 20$

(b) When $f(x) = 0$

$x^3 - 9x^2 + 24x - 20 = 0$

Use GC, $x = 2$ or 5

When $f(|x|) = 0$

$\therefore\quad |x| = 2 \quad \text{or} \quad |x| = 5$ $\quad\triangleleft$ replacing x by $|x|$

$\qquad x = \pm 2 \quad \text{or} \quad |x| = \pm 5$

The roots of $f(|x|) = 0$ are $\pm 2,\ \pm 5$.

57
Solution

Given $x^2 + Ay^2 + By + C = 0$ .... (1)

Substitute (2.15, 0) into (1)

$0^2 + A(0)^2 + B(0) + C = 0$ .... (2)

$C = 2.15^2$

$= -4.6225$

Substitute (1, 6.8) into (1)

$6.8^2 A + 6.8B + C = 0$

$46.24A + 6.8B = 3.6225$ .... (3)

Substitute (0, 7) into (1)

$40A + 7B + C = 0$ .... (4)

Substitute $C = -4.6225$ into (2)

$49A + 7B = 4.6225$ .... (5)

Using GC to solve (3) and (5)

$A = 0.6381827731, B = -3.806922269$

$\therefore\ A = 0.6382,\ B = -3.8069$ and $C = -4.6225$

Substitute $A = 0.6382,\ B = -3.8069$ and $C = -4.6225$ into (1)

$x^2 + 0.63818y^2 - 3.8069y - 4.6225 = 0$ ⊲ express the equation of ellipse of the form $\frac{(x-h)^2}{a^2} + \frac{(x-k)^2}{b^2} = 1$

$x^2 + 0.63818(y^2 - 5.9653y) - 4.6225 = 0$

$x^2 + 0.63818[(y - 2.9826)^2 - 8.8961] - 4.6225 = 0$

$x^2 + 0.63818(y - 2.9826)^2 = 10.300$

When $y = 2.9826$,

$x^2 + 0.63818(2.9826 - 2.9826)^2 = 10.300$

$x^2 = 10.300$

$\therefore \quad x = 3.2093$

The widest part of tunnel is 6.4 cm (correct to nearest 0.1)

58
Solution

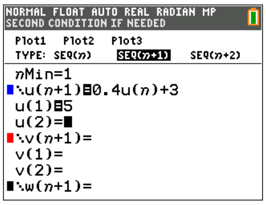
(a) $u_2 = 0.4u_1 + 3$

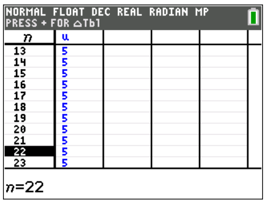
$u_3 = 0.4u_2 + 3$

$u_4 = 0.4u_3 + 3$

$\therefore\ u_{n+1} = 0.4u_n + 3$ (Shown)

(b) Use GC to determine the behavior of the number of bacteria in culture A in the long run.





The number of bacteria in culture A remains constant at 5 million.

(c) Given $v_n = \dfrac{pn}{n^2 + qn + r}$ .... (1)

At the start of the 1st day, i.e. $n = 1$.
When $n = 1$, $v_1 = 2$.
Substitute $n = 1$ and $v_1 = 2$ into (1)

$$2 = \frac{p}{1 + q + r}$$

$2 + 2q + 2r = p$

$p - 2q - 2r = 2$ .... (2)

At the start of the 2nd day, i.e. $n = 2$.
When $n = 2$, $v_2 = 2.4$.
Substitute $n = 1$ and $v_1 = 2$ into (1)

$$2.4 = \frac{2p}{4 + 2q + r}$$

$9.6 + 4.8q + 2.4r = 2p$

$p - 2.4q - 1.2r = 4.8$ .... (3)

At the start of the 4th day, i.e. $n = 4$.
When $n = 4$, $v_2 = 1.6$.
Substitute $n = 4$ and $v_1 = 1.6$ into (1)

$$1.6 = \frac{4p}{16 + 4q + r}$$

$25.6 + 6.4q + 1.6r = 4p$

$p - 1.6q - 0.4r = 6.4$ .... (4)

Use GC to solve (2), (3) and (4)

$\therefore\ p = 6, q = -1, r = 3.$

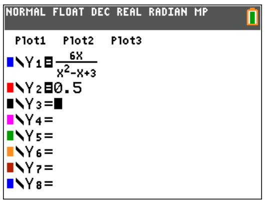
(d) Substitute $p = 6$, $q = -1$ and $r = 3$ into $v_n = \dfrac{pn}{n^2 + qn + r}$.

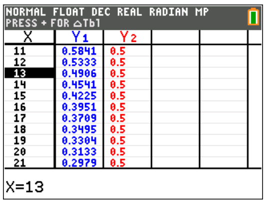
When the number of bacteria in culture B first fall below half a million,

i.e. $$v_n < 0.5$$

$$\frac{6n}{n^2 - n + 3} < 0.5$$

Using GC





X	Y1	Y2
11	0.5841	0.5
12	0.5333	0.5
13	0.4906	0.5
14	0.4541	0.5
15	0.4225	0.5
16	0.3951	0.5
17	0.3709	0.5
18	0.3495	0.5
19	0.3304	0.5
20	0.3133	0.5
21	0.2979	0.5

From table, least $n = 13$

Therefore, the researcher first record the number of bacteria in culture B to be below half a million on 13 April.

Exercise 4

I Higher Order Questions

59
Solution

Given $(10-x)(10-|x|)>11$

Case 1: When $x<0$, i.e. $|x|=-x$

$$(10-x)(10+x)>11$$
$$100-x^2>11$$
$$89-x^2>0$$
$$x^2-89<0$$
$$(x-\sqrt{89})(+x+\sqrt{89})>0$$
$$-\sqrt{89}<x<\sqrt{89}$$

Since $x<0$, then $-\sqrt{89}<x<0$ .... (1)

Case 2: When $x\geq 0$, i.e. $|x|=x$

$$(10-x)(10-x)>11$$
$$100-20x+x^2>11$$
$$x^2-20x+89>0$$

Let $x^2-20x+89=0$

$$x=\frac{20\pm\sqrt{400-356}}{2}$$
$$=10\pm\sqrt{11}$$

For $x^2-20x+89>0$

$x<10-\sqrt{11}$ or $x>10+\sqrt{11}$

Since $x\geq 0$, then $0\leq x<10-\sqrt{11}$ .... (2)

Combining (1) and (2)

$\therefore\ -\sqrt{89}<x<10-\sqrt{11}$ or $x>10+\sqrt{11}$

Alternative Method

Given $(10-x)(10-|x|)>11$

For $x<0$, $|x|=-x$

$\therefore \quad (10-x)(10+x)>11$

$100-x^2>11$

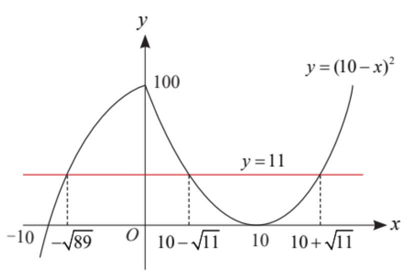
Sketch $y=100-x^2$ for $x<0$

When $x \geq 0, |x|=x$

$(10-x)(10-x)>11$

$(10-x)^2>11$

Sketch $y=(10-x)^2$ for $x \geq 0$



Find the points of intersection, for $x<0$

$100-x^2=11$

$x=-\sqrt{89}$ or $x=\sqrt{89}$ (rejected)

Find the points of intersection, for $x \geq 0$

$(10-x)^2=11$

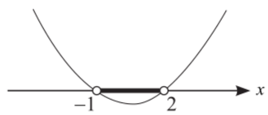
$$x=\frac{20 \pm \sqrt{400-356}}{2}$$

$=10 \pm \sqrt{11}$

From the graph, $\sqrt{89}<x<10-\sqrt{11}$ or $x>10+\sqrt{11}$

60
Solution

(a)
$$\frac{2x-1}{x-2}<1$$
$$\frac{2x-1}{x-2}-1<0,\ x\in\mathbb{R},\ x\neq 2.$$
$$\frac{2x-1-(x-2)}{x-2}<0$$
$$\frac{x+1}{x-2}<0 \quad \triangleleft \text{ multiply by } (x-2)^2 \text{ on both sides}$$
$$(x+1)(x-2)<0$$
$\therefore\ -1<x<2$



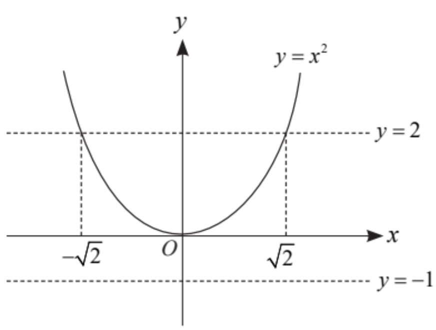
(b)(i) $\mathrm{f}(x^2)<1$

Use result **(a)** and replace x with x^2

$\therefore\ -1<x^2<2$

Using graphical Method

Graph $y=x^2$, $y=-1$ and $y=2$.



Refer to the above diagram.

For $-1<x^2<2$, the range of values of x is $-\sqrt{2}<x<\sqrt{2}$.

$\therefore\ -\sqrt{2}<x<\sqrt{2}$

Using Analytical Method

Use result **(a)** and replace x with x^2

$\therefore\ -1<x^2<2$

$x^2>-1$ and $x^2<2$

$x\in\mathbb{R}$ and $(x+\sqrt{2})(x-\sqrt{2})<0$

$-\sqrt{2}<x<\sqrt{2}$

$\therefore\ -\sqrt{2}<x<\sqrt{2}$

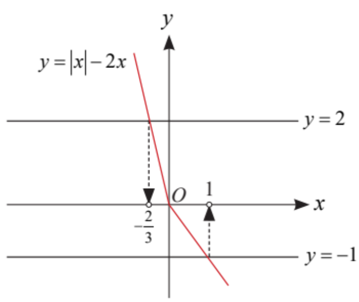
(b)(ii) Using graphical Method

Given $f(|x|-2x)<1$

Replace x with $|x|-2x$ in **(b)(i)**:

i.e. $-1<|x|-2x<2$

Sketch $y=|x|-2x$, $y=-1$ and $y=2$.



Refer to the graphs.

$-1<|x|-2x<2$, the range of values of x is $-\dfrac{2}{3}<x<1$.

$\therefore\ -\dfrac{2}{3}<x<1$

‘

61
Solution

(a) Given $a > 0$

$$a+\frac{1}{4} > \frac{1}{4} \qquad \triangleleft \text{ add 4 on both sides}$$

$$\sqrt{a+\frac{1}{4}} > \sqrt{\frac{1}{4}} \qquad \triangleleft \text{ square root on both sides}$$

$$\sqrt{a+\frac{1}{4}} > \frac{1}{2}$$

$$-\sqrt{a+\frac{1}{4}} < -\frac{1}{2}$$

$$\frac{3}{2}-\sqrt{a+\frac{1}{4}} < \frac{3}{2}-\frac{1}{2} \qquad \triangleleft \text{ add } \frac{3}{2} \text{ on both sides}$$

$$\therefore \ \frac{3}{2}-\sqrt{a+\frac{1}{4}} < 1 \ \text{ (Shown)}$$

(b) $$\frac{x^2-x-a}{x-1} \le 2 \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (1)$$

$$\frac{x^2-x-a}{x-1}-2 \le 0$$

$$\frac{x^2-x-a-2x+2}{x-1} \le 0$$

$$\frac{x^2-3x+2-a}{x-1} \le 0$$

$$\frac{\left(x-\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2+2-a}{x-1} \le 0 \qquad \text{completing the square in the numerator}$$

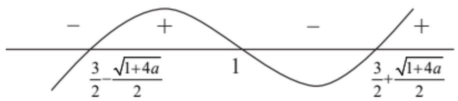
$$\frac{\left(x-\frac{3}{2}\right)^2-\left(\frac{1}{4}+a\right)}{x-1} \le 0$$

$$\frac{\left(x-\frac{3}{2}\right)^2-\left(\sqrt{\frac{1+4a}{4}}\right)^2}{x-1} \le 0$$

$$\frac{\left(x-\frac{3}{2}+\frac{\sqrt{1+4a}}{2}\right)\left(x-\frac{3}{2}-\frac{\sqrt{1+4a}}{2}\right)}{x-1} \le 0$$

$$\frac{\left[x-\left(\frac{3}{2}-\sqrt{\frac{1}{4}+a}\right)\right]\left[x-\left(\frac{3}{2}+\sqrt{\frac{1}{4}+a}\right)\right]}{x-1} \le 0$$

From **(a)**, $\left(\frac{3}{2}-\sqrt{\frac{1}{4}+a}\right)<1$, where a is a positive constant. Hence, $\left(\frac{3}{2}+\sqrt{\frac{1}{4}+a}\right)>\frac{3}{2}>1$.



$\therefore\ x\le\frac{3}{2}-\frac{\sqrt{1+4a}}{2}$ or $1<x\le\frac{3}{2}+\frac{\sqrt{1+4a}}{2}$ .... (*)

(c) To solve the inequality $\frac{a+x-x^2}{x}\le 2$.

Replace x with $1-x$ in (1):

$$\frac{(1-x)^2-(1-x)-a}{(1-x)-1}\le 2$$

$$\frac{1-2x+x^2-1+x-a}{-x}\le 2$$

$$\frac{x^2-x-a}{-x}\le 2$$

$$\frac{a+x-x^2}{x}\le 2$$

Deduce the result in (*). Replace x with $(1-x)$ in (*).

$1-x\le\frac{3}{2}-\frac{\sqrt{1+4a}}{2}$ or $1<1-x\le\frac{3}{2}+\frac{\sqrt{1+4a}}{2}$

$\therefore\ x\ge-\frac{1}{2}+\frac{\sqrt{1+4a}}{2}$ or $-\frac{1}{2}-\frac{\sqrt{1+4a}}{2}\le x<0$

62
Solution

(a) Given $y = ax + \dfrac{b}{x-2}$

$$\frac{dy}{dx} = a - \frac{b}{(x-2)^2}$$

For turning points, $\dfrac{dy}{dx} = 0$.

$$a - \frac{b}{(x-2)^2} = 0$$

$$(x-2)^2 = \frac{b}{a}$$

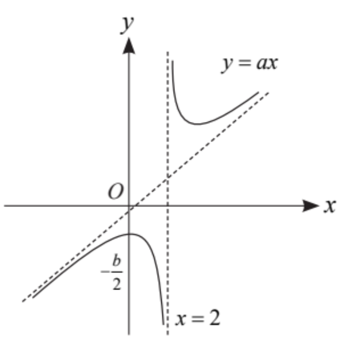
$$x = 2 \pm \sqrt{\frac{b}{a}}$$

$\therefore$ a and b must have the same sign for two turning points to exist.

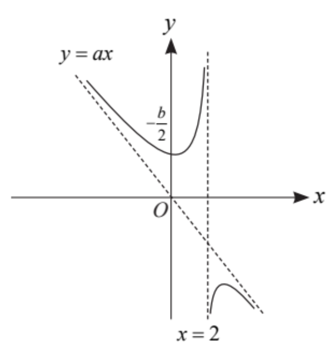
Both a and b are positive or both a and b are negative for the curve C to have two turning points.

Learning point:

The graph of $y = ax + \dfrac{b}{x-2}$, where $a, b > 0$.



The graph of $y = ax + \dfrac{b}{x-2}$, where $a, b > 0$.



(b) Given that $a = b$,

$\therefore\ y = ax + \dfrac{a}{x-2}$

$y(x-2) = ax(x-2) + a$

$ax^2 - (2a + y)x + (a + 2y) = 0$ .... (1)

For y can take, equation (1) must have real roots.

$\therefore$ Discriminant ≥ 0

$$(2a + y)^2 - 4a(a + 2y) \geq 0$$

$$4a^2 + 4ay + y^2 - 4a^2 - 8ay \geq 0$$

$$y^2 - 4ay \geq 0$$

$$y(y - 4a) \geq 0$$

If $a > 0$, then $y \leq 0$ or $y \geq 4a$.
If $a < 0$, then $y \leq 4a$ or $y \geq 0$.

Alternative Method

From **(a)**: $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2 \pm \sqrt{\dfrac{b}{a}}$

Given that $a = b$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2 \pm \sqrt{\dfrac{a}{a}}$.

At stationary point, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$

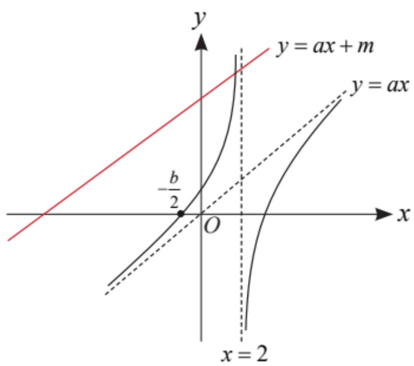
$\therefore \quad x = 2 \pm \sqrt{\dfrac{b}{a}}$

$\quad = 1$ or 3.

When $x = 1$, $y = 0$ and when $x = 3$, $y = 4a$.

If $a > 0$, then $y \le 0$ or $y \ge 4a$.
If $a < 0$, then $y \le 4a$ or $y \ge 0$.

(c) The graphs of $y = ax + \dfrac{b}{x-2}$ and $y = ax + m$.



Refer to the diagram above. For the inequality $ax + \dfrac{b}{x-2} < ax + m$ has the solution set $\{x \in \mathbb{R} : x < 0 \text{ or } x > 2\}$,

the line $y = ax + m$ lies above x-intercept of the curve $y = ax + \dfrac{b}{x-2}$.

$\therefore \ m = -\dfrac{b}{2}$

63
Solution

(a) Given $b|x-a|=x-ab,$ where $a<0$ and $b<-1$.

Consider $x-a\geq 0$.

$$x-a=\frac{x}{b}-a$$

$$x-\frac{x}{b}=0$$

$$x=0$$

Consider $x-a<0$.

$$-(x-a)=\frac{x}{b}-a$$

$$a-x=\frac{x}{b}-a$$

$$\frac{x}{b}+x=2a$$

$$x=\frac{2ab}{b+1}$$

$\therefore$ the roots of the equation are $\frac{2ab}{b+1}$ and 0.

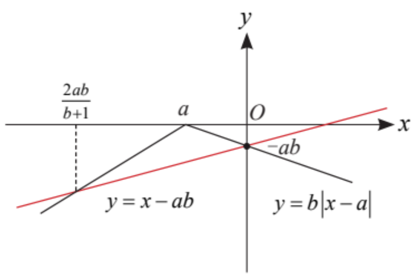
(b)(i) To have a negative root, $x=\frac{2ab}{b+1}<0.$

Given $a<b<0,$ then $ab>0.$

$\therefore \quad b+1<0$

$b<-1$

(b)(ii) The graphs of $y = x - ab$ and $y = b|x-a|$.



For $b|x-a| \geq x - ab,$ from the graph above, $\dfrac{2ab}{b+1} \leq x \leq 0.$ .... (1)

Hence the required inequality is $\dfrac{2ab}{b+1} \leq x \leq 0.$

(b)(iii) To solve $-b|e^x + a| \leq e^x - ab,$

from **(i)** $\quad b|x-a| \geq x - ab \quad$ ⊲ replace x with e^x

$$b|e^x - a| \leq e^x - ab \quad \text{⊲ multiply negative on both sides}$$

$$-b|e^x + a| \geq -e^x - ab$$

$$b|-(e^x + a)| \geq -e^x - ab$$

$$b|-e^x - a)| \geq -e^x - ab$$

Deduce the result in (1). Replace x with $-e^x$ in (1).

$$\frac{2ab}{b+1} \leq -e^x < 0$$

$$0 < e^x \leq -\frac{2ab}{b+1} \quad \text{⊲ multiply negative on both sides}$$

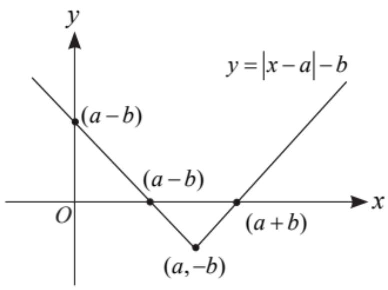
$$\therefore \quad e^x \leq -\frac{2ab}{b+1}$$

$$x \leq \ln\left(-\frac{2ab}{b+1}\right)$$

$$\therefore \; x \leq \ln\left(-\frac{2ab}{b+1}\right)$$

64
Solution

(a) The graph of $y = |x - a| - b$.



(b) $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$

$\downarrow$ (Replace x by $x-a$)

$\dfrac{(x-a)^2}{a^2}+\dfrac{y^2}{b^2}=1$

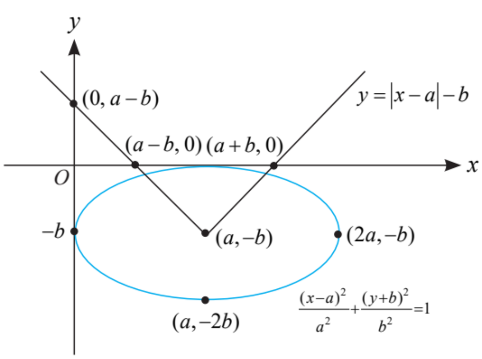
$\downarrow$ (Replace y by $y+b$)

$\dfrac{(x-a)^2}{a^2}+\dfrac{(y+b)^2}{b^2}=1$

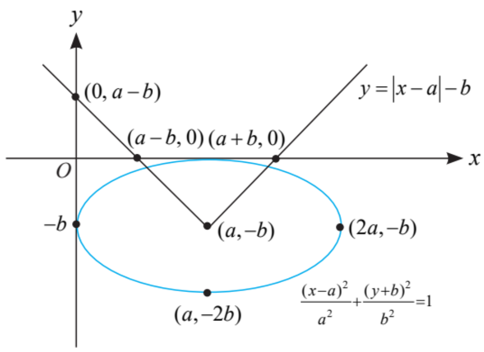
Description of sequence of transformations

A translation of magnitude a units in the positive direction of the x-axis.

A translation of magnitude b units in the negative direction of the y-axis.



(c)



Observe $y = -b + \frac{b}{a}\sqrt{a^2 - (x-a)^2}$ in the inequality $(|x-a|-b)\left(-b + \frac{a}{b}\sqrt{a^2-(x-a)^2}\right) < 0.$

Rewrite $y = -b + \frac{b}{a}\sqrt{a^2 - (x-a)^2}$ as $\frac{(x-a)^2}{a^2} + \frac{(y+b)^2}{b^2} = 1.$

Thus $y = -b + \frac{b}{a}\sqrt{a^2 - (x-a)^2}$ is the top half of the ellipse, and is in the 4th quadrant.

Hence $y = -b + \frac{b}{a}\sqrt{a^2 - (x-a)^2} \le 0$ for $0 \le x \le 2a.$

For $(|x-a|-b)\left(-b + \frac{b}{a}\sqrt{a^2-(x-a)^2}\right) < 0,$ it is given that the solution set is $0 \le x > 1$ or $5 < x \le 2a.$

$\therefore\ y = |x-a| - b > 0.$

From the diagram,

$$a - b = 1 \text{ .... } (1)$$

$$a + b = 5 \text{ .... } (2)$$

Solve (1) and (2) : $a = 3,\ b = 2$

$\therefore\ a = 3$ and $b = 2$

65
Solution

(a) Let the number of days Li Yang spent in Brisbane, Melbourne and Perth be x, y, z respectively.

$$430x + 250y + 230 = 1560 \text{ .... } (1)$$

$$100x + 120y + 150 = 390 \text{ .... } (2)$$

$$90x + 100y + 120 = 350 \text{ .... } (3)$$

Using GC to solve (1), (2) and (3) : $x = 3,\ y = 2,\ z = -1$

Since $z = -1$ which is negative, his records are incorrect.

(b) $430x + 250y + 230z = 1590$ .... (3)

$x + y + z = 5$ .... (4)

Using GC to solve (3) and (4) :

$$x = \frac{17}{9} + \frac{1}{9}\lambda$$

$$y = \frac{28}{9} - \frac{10}{9}\lambda$$

$$z = \lambda$$

$x,\ y,\ z \in \mathbb{Z}^+$. by eliminations, $\lambda = 1$ is the only possible answer.

Thus, $x = 2,\ y = 2,\ z = 1$

Total amount he spent in Brisbane

$= \$430 \times 2 + \$100 \times 2 + \$90 \times 2$

$= \$1240$

66
Solution

Let $\$x$ and $\$y$ be the price of a small mug and a big mug respectively.

Amount paid by retailer A: $3x + 2y = 46.50$ .... (1)

Amount paid by retailer B: $kx + 3ky = 217.50$

$$x + 3y - 217.50\left(\frac{1}{k}\right) = 0 \text{ .... (2)}$$

Amount paid by retailer C: $kx + 2ky = 157.50$

$$x + 2y - 157.50\left(\frac{1}{k}\right) = 0 \text{ .... (3)}$$

Using GC to solve (1), (2) and (3) : $x = \$7.50,\ y = \$12,\ \frac{1}{k} = \frac{1}{5}$

Hence, $k = 5$.

The price of each small mug is \$7.50 and the price of each large mug is \$12.
The value of k is 5.

67
Solution

Let A be the number of appetizer purchased
Let B be the number of main dish purchased
Let C be the number of the dessert purchased

$$A+B+C=83 \text{ .... (1)}$$

Scenario 1: If $26 \le A \le 30$ are ordered, A has no discount.

$$0A+0.1(7)B+0.05(2)C=\$21.7 \text{ .... (2)}$$

$$(1.5)A+0.9(7)B+0.95(2)C=\$265.3 \text{ .... (3)}$$

Using GC to solve (1), (2) and (3): $A=28,\ B=27,\ C=28$

Scenario 2: If $A>30$ are ordered, A has discount.

$$0.15(1.5A)+0.1(7)B+0.05(2)C=\$21.7 \text{ .... (4)}$$

$$0.85(1.5A)+0.9(7)B+0.95(2)C=\$265.3 \text{ .... (5)}$$

Using GC to solve (1), (4) and (5): $A=-6.05,\ B=23.59,\ C=65.46$ (NA)

He ordered 28 servings of the appetizer, 27 serving of main dish and 28 serving of dessert.

68
Solution

Total number of books bought:

$$x + y + z = 43 \text{ .... (1)}$$

Total selling price of the books before discount:

$$11.6x^2 + 9.3y^2 + 13.7z^2 = 7921$$

$$\frac{11.6x^2 + 9.3y^2 + 13.7z^2}{100} = 79.21 \text{ .... (2)}$$

Total amount paid after discounted :

$$x\left(1 - \frac{x}{100}\right)(11.60) + y\left(1 - \frac{y}{100}\right)(9.30) + z\left(1 - \frac{z}{100}\right)(13.70) = 429.59 \text{ .... (3)}$$

$$11.6x + 9.3y + 13.7z - \frac{11.6x^2 + 9.3y^2 + 13.7z^2}{100} = 429.59$$

Substitute (2) into (3)

$$11.6x + 9.3y + 13.7z - 79.21 = 429.59$$

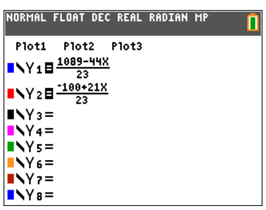
$$11.6x + 9.3y + 13.7z = 508.80 \text{ .... (4)}$$

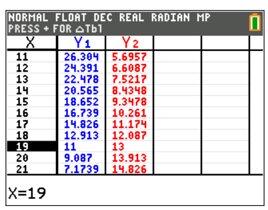
Using GC to solve (1) and (4)

$$x = \frac{1089 - 44z}{23}$$

$$y = \frac{-100 + 21z}{23}$$

Since x, y and z are integers, use the GC to check values





Form the GC, $x = 11$, $y = 13$, $z = 19$.

She bought 11 mathematics textbooks, 13 physics textbooks and 19 chemistry textbooks.

69
Solution

(a) Option A

$5(10-k)x+4kx+5(3)y=1455$

$50x-kx+15y=1455$ .... (1)

Option B

$5(10)x+5(3)y=1500$

$50x+15y=1500$

$y=100-\dfrac{10}{3}x$ .... (2)

$(2)-(1): kx=45$

As x is a multiple of 5, k can only be 1, 3 or 9

When $k=1$, $x=45$,

$$y=100-\frac{10}{3}(45)$$
$$=-50 \quad \text{(rejected since } y \text{ is positive)}$$

When $k=3$, $x=15$,

$$y=100-\frac{10}{3}(15)$$
$$=50$$

When $k=9$, $x=5$,

$$y=100-\frac{10}{3}(5)$$
$$=83\frac{1}{3} \quad \text{(reject since } y \text{ is number that is multiple of 5)}$$

$\therefore\ x=15,\ y=50$

Exercise 4

J Exam Style Questions

70
Solution

$$x^2(x-5) \geq (x-5)(2kx-k^2)$$
$$(x-5)[x^2-(2kx-k^2)] \geq 0$$
$$(x-5)(x^2-2kx+k^2) \geq 0$$
$$(x-5)(x-k)^2 \geq 0$$

$\therefore\ x = k$ or $x \geq 5$.

71
Solution

$$\frac{4x^2-x-1}{(2x-1)(x+1)} \geq 1 \text{ .... } (1)$$

$$\frac{4x^2-x-1}{(2x-1)(x+1)} - 1 \geq 0$$

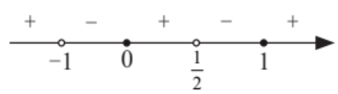
$$\frac{(4x^2-x-1)-(2x-1)(x+1)}{(2x-1)(x+1)} \geq 0$$

$$\frac{(4x^2-x-1)-(2x^2+x-1)}{(2x-1)(x+1)} \geq 0$$

$$\frac{2x^2-2x}{(2x-1)(x+1)} \geq 0$$

$$\frac{2x(x-1)}{(2x-1)(x+1)} \geq 0 \text{ .... } (*)$$

$$2x(x-1)(2x-1)(x+1) \geq 0$$



$\therefore\ x < -1$ or $0 \leq x < \frac{1}{2}$ or $x \geq 1$

To solve the inequality $\frac{4e^{2x}-e^x-1}{(2e^x-1)(e^x+1)} \geq 1$

Replace x with e^x in (1):

$$\frac{4e^{2x}-e^x-1}{(2e^x-1)(e^x+1)} \geq 1$$

Deduce the result in (*). Replace x by e^x in (*).

For $e^x < -1$ (no real solution)

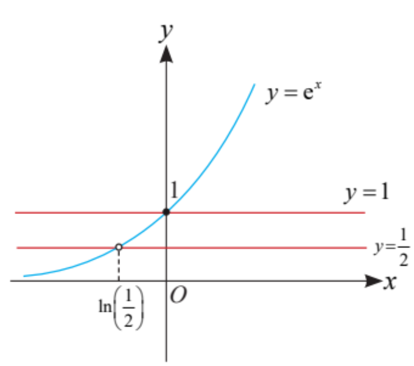
For $0 \leq e^x < \frac{1}{2}$

$x < \ln\frac{1}{2}$

For $e^x \geq 1$

$x \geq 0$

From the diagram, $x < \ln\frac{1}{2}$ or $x \geq 0$.



72
Solution

(a) $3x-x^2-4=-(x^2-3x+4)$

$$=-\left(\left(x-\frac{3}{2}\right)^2+\frac{7}{4}\right)$$

$$=\left(x-\frac{3}{2}\right)^2-\frac{7}{4}$$

Since $\left(x-\frac{3}{2}\right)^2\geq 0$ for all $x\in\mathbb{R}$, $-\left(x-\frac{3}{2}\right)^2\leq 0$

Hence $3x-x^2-4=-\left(x-\frac{3}{2}\right)^2-\frac{7}{4}\leq-\frac{7}{4}<0$

$\therefore$ $3x-x^2-4$ is always negative for all values of x.

(b) $\dfrac{(3x-x^2-4)(x-1)^2}{x^2-2x-5}\leq 0$

$3x-x^2-4$ is always negative deduced in **(a)**

$\therefore$ $\dfrac{(x-1)^2}{x^2-2x-5}\geq 0$

Let $x^2-2x-5=0$

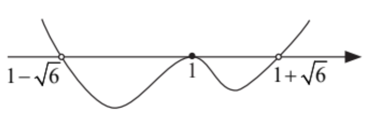
$$\therefore\quad x=\frac{2\pm\sqrt{(-2)^2-4(1)(-5)}}{2(1)}$$

$$=\frac{2\pm\sqrt{24}}{2}$$

$$=1\pm\sqrt{6}$$

Hence $\dfrac{(x-1)^2}{(x-(1-\sqrt{6}))(x-(1+\sqrt{6}))}\geq 0$

$(x-1)^2(x-(1-\sqrt{6}))(x-(1+\sqrt{6}))\geq 0$

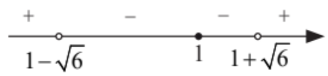


$\therefore$ $x<1-\sqrt{6}$ or $x>1+\sqrt{6}$ or $x=1$

73
Solution

(a) $\dfrac{2x^2-4x+11}{x+1} \geq x+1,\ x \neq -1$ .... (1)

$\dfrac{2x^2-4x+11}{x+1} - x - 1 \geq 0$



$\dfrac{x^2-6x+10}{x+1} \geq 0$

Consider $x^2-6x+10$

By completing the square, $x^2-6x+10 = (x-3)^2+1$

Since $(x-3)^2 \geq 0,$ therefore $(x-3)^2+1 > 0$ for all real values of x.

$\therefore\ \dfrac{1}{x+1} \geq 0$

$(x+1) \geq 0$

$x \geq -1$

The set of values is $\{x \in \mathbb{R} : x > -1\}$

(b) To solve the inequality $2x^4-4x^2+11 \geq (x^2+1)^2$

Replace x with x^2 in (1): $\dfrac{2x^4-4x^2+11}{x^2+1} \geq x^2+1$

$\therefore\quad 2x^4-4x^2+11 \geq (x^2+1)^2$

Deduce the result in (*). Replace x with (x^2) in (*).

$x^2 > -1$

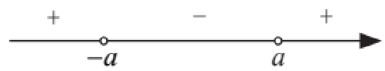
Since $x^2 \geq 0$ for all real values of $x,$ the solution set is $\{x : x \in \mathbb{R}\}$.

74
Solution

(a) **Method 1**: **Completing the square**

$$x^2 - 2ax + b = (x-a)^2 - a^2 + b$$
$$= (x-a)^2 + (b-a^2)$$

Since $(x-a)^2 \geq 0$ and given that $b > a^2$, i.e. $b - a^2 > 0$.



$\therefore$ $x^2 - 2ax + b$ is always positive for all real values of x

Method 2: **Discriminant**

Since the coefficient of $x^2 = 1$ is positive, and

$$\text{Discriminant} = (-2a)^2 - 4(1)(b)$$
$$= 4a^2 - 4b$$
$$= 4(a^2 - b)$$

Thus $4(a^2 - b) < 0$ since $b > a^2$

$\therefore$ $x^2 - 2ax + b$ is always positive for all real values of x.

(b) $\dfrac{x^2 - 2ax + 2a^2}{x^2 - a^2} \leq 0$ .... (1)

Hence method:

From part **(a)**, $x^2 - 2ax + 2a^2$ is also always positive for all real values of x.

Hence $\quad \dfrac{1}{x^2 - a^2} < 0$

$$\frac{1}{(x+a)(x-a)} < 0$$

$$(x+a)(x-a) < 0$$

$\therefore$ $-a < x < a$ .... (*)

Otherwise method : **Completing the square**

$x^2 - 2ax + 2a^2$

$= (x-a)^2 - a^2 + 2a^2$

$= (x-a)^2 + a^2$

$(x-a)^2 \geq 0$, for all $x \in \mathbb{R}$

$\therefore$ $(x-a)^2 + a^2 > 0$

$x^2 - 2ax + 2a^2$ is always positive for all real values of x.

So, $\dfrac{1}{x^2 - a^2} < 0$

$$\frac{1}{(x+a)(x-a)} < 0$$

$$(x+a)(x-a) < 0$$

$\therefore$ $-a < x < a$

(c) To solve the inequality $\dfrac{x^4-2ax^2+2a^2}{x^4-a^2}\le 0$

Replacing x with x^2 in (1): $\dfrac{x^2-2ax+2a^2}{x^2-a^2}\le 0$

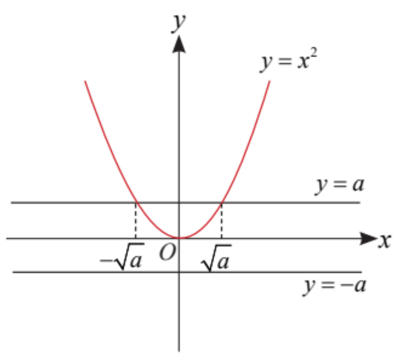
$\therefore \quad \dfrac{x^4-2ax^2+2a^2}{x^4-a^2}<0$

Deduce the result in (*). Replace x with (x^2) in (*).

$\therefore \ -a<x^2<a$

Method 1: Using graphical approach

Sketch $y=x^2$, $y=a$ and $y=-a$



From the graph above, to satisfy the inequality $-a<x^2<a$

$\therefore \ -\sqrt{a}<x<\sqrt{a}$

Method 2: Using algebraic approach

From $-a<x^2<a$

$x^2>-a$ and $x^2<a$

$x\in\mathbb{R}$ and $x^2-a<0$

$(x+\sqrt{a})(x-\sqrt{a})<0$

$\therefore \ -\sqrt{a}<x<\sqrt{a}$

75
Solution

(a) $\left|2x^2+3x-2\right|=2-x$

$2x^2+3x-2=2-x$ or $2x^2+3x-2=-(2-x)$

Consider $2x^2+3x-2=2-x$

$\therefore \quad 2x^2+4x-4=0$

$x^2+2x-2=0$

$$x=\frac{-2\pm\sqrt{4+8}}{2}$$
$$=-1\pm\sqrt{3}$$

and then consider, $2x^2+3x-2=-(2-x)$

$\therefore \quad 2x^2+2x=0$

$x(x+1)=0$

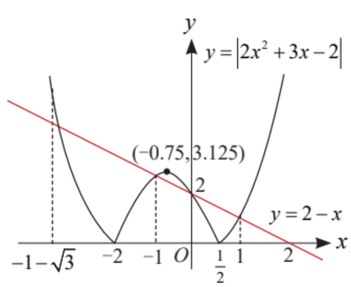
$x=0$ or -1

Substitute $x=0,\ -1,\ -1+\sqrt{3},\ -1-\sqrt{3}$ into the equation $\left|2x^2+3x-2\right|=2-x$.

Since all $x=0,\ -1, -1+\sqrt{3},\ -1-\sqrt{3}$ satisfy the equation $\left|2x^2+3x-2\right|=2-x$, all solutions are accepted.

$\therefore\ x=0, -1,\ -1+\sqrt{3},\ -1-\sqrt{3}$

(b) Sketch $y=\left|2x^2+3x-2\right|$ and $y=2-x$.

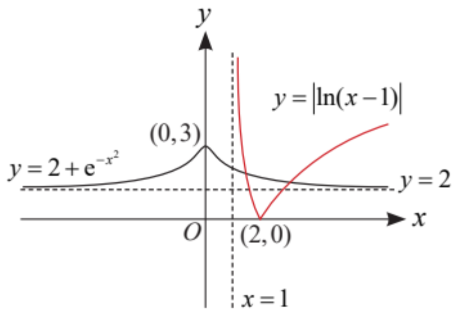


From graph, when $\left|2x^2+3x-2\right|<2-x$,

$-1-\sqrt{3}<x<-1$ or $0<x<-1+\sqrt{3}$.

76
Solution

(a) The graphs of $y = 2 + e^{-x^2}$ and $y = |\ln(x-1)|$.

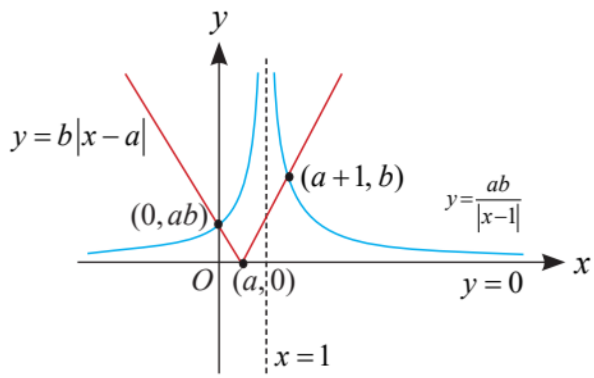


(b) Using graphic calculator to find x-coordinates of points of intersection.
$\therefore\ x = 1.10$ and 8.39

From the graph, for $2 + e^{-x^2} \le |\ln(x-1)|$,
$1 < x \le 1.10$ or $x \ge 8.39$ (3 s.f.)

77
Solution

(a) The graphs of $y = \dfrac{ab}{|x-1|}$ and $y = b|x-a|$, where $0 < a < 1 < b$.



(b) To find the point of intersection between the graphs,
consider $x > 1$

$$b(x-a) = \frac{ab}{x-1}$$

$$(x-a) = \frac{a}{x-1}$$

$$x^2 - x - ax + a = a$$

$$x^2 - x - ax = 0$$

$$x(x-1-a) = 0$$

$$x = a+1 \quad \text{or} \quad x = 0 \ \text{(Rejected, since } x > 1\text{)}$$

Consider $x < 1$

$$-b(x-a) = -\frac{ab}{x-1}$$

$$(x-a) = \frac{a}{x-1} \qquad \triangleleft \text{ use the above result}$$

$$x = 0 \quad \text{or} \quad x = a+1 \ \text{(Rejected, since } x < 1\text{)}$$

Given $\quad |x-a| \le \dfrac{a}{|x-1|} \qquad \triangleleft$ multiplying b on both sides

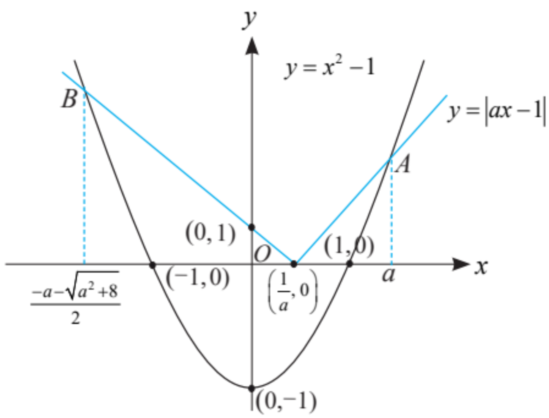
$$b|x-a| \le \frac{ab}{|x-1|} \qquad \triangleleft \text{ since } b > 0$$

Hence from the graph, for $|x-a| \le \dfrac{b}{|x-1|}$

$\therefore \ 0 \le x \le a+1, \ x \ne 1$

78
Solution

(a) The graphs of $y = |ax-1|$, where $a > 1$, and $y = x^2 - 1$



From the graph, at A

$$ax - 1 = x^2 - 1$$
$$x^2 - ax = 0$$
$$x(x-a) = 0$$
$$x = 0 \text{ (rejected since } x > 0) \quad \text{or} \quad x = a$$

From the graph, at B

$$-(ax-1) = x^2 - 1$$
$$x^2 + ax - 2 = 0$$
$$x = \frac{-a-\sqrt{a^2+8}}{2} \quad \text{or} \quad \frac{-a+\sqrt{a^2+8}}{2} \text{ (Rejected since } x < 0)$$

From the graph, for $|ax-1| > x^2 - 1$, the solution set is $\left\{x : x \in \mathbb{R},\ \frac{-a-\sqrt{a^2+8}}{2} < x < a\right\}$ .... (1)

(b) To solve the inequality $\left|a^{x+1} - 1\right| > a^{2x} - 1$

Replacing x with a^x in $|ax-1| > x^2 - 1$: $\left|a(a^x) - 1\right| > (a^x)^2 - 1$.

$\therefore \quad \left|a^{x+1} - 1\right| > a^{2x} - 1$

Deduce the result in (*). Replace x with (a^x) in (*).

$$\therefore \quad \frac{-a-\sqrt{a^2+8}}{2} < a^x < a$$
$$\frac{-a-\sqrt{a^2+8}}{2} < a^x \quad \text{and} \quad a^x < a$$
$$x \in \mathbb{R} \quad \text{and} \quad x < 1$$

Since $\frac{-a-\sqrt{a^2+8}}{2} < 0 < a^x$ for all real x, so $x < 1$

The solution set is $\{x : x \in \mathbb{R},\ x < 1\}$

79
Solution

(a) $y = \dfrac{x^2 + ax + b}{x+9}$

Perform long division,

$$\begin{array}{r}
x + (a-9) \\
x+9 \overline{\big) x^2 + ax + b } \\
\underline{-(x^2 + 9x) } \\
(a-9)x + b \\
\underline{-\left[(a-9)x + (a-9)9\right]} \\
81 - 9a + b
\end{array}$$

$y = x + (a-9) + \dfrac{81 - 9a + b}{x+9}$

$\therefore$ vertical asymptote is $x = -9$ and the oblique asymptote is $y = x + (a-9)$.

The two asymptotes intersect at $(-9, -12)$.

Substitute $x = -9,\ y = -12$ into $y = x + (a-9)$.

$\therefore \quad -12 = -9 + (a-9)$

$a = 6$

When the curve cuts the x-axis, i.e. $y = 0$.

Substitute $y = 0$ into $y = \dfrac{x^2 + ax + b}{x+9}$.

$\therefore \quad x^2 + 6x + b = 0$

Given that curve cuts the x-axis only once, discriminant $= 0$

Using discriminant, $6^2 - 4(1)b = 0$

$\therefore \ b = 9$

$\therefore \ a = 6$ and $b = 9$.

(b) $y=\dfrac{x^2+6x+9}{x+9}$

$=x-3+\dfrac{36}{x+9}$

Equations of asymptotes

$x=3$ is a vertical asymptote.

$y=2x+6$ is a horizontal asymptote.

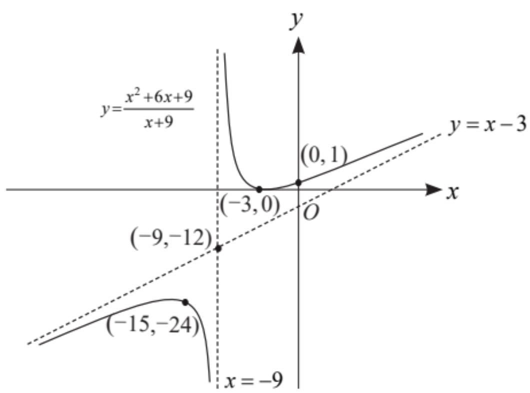
Axial intercept : When $x=0, y=1$

When $y=0, x=-3$

Use G.C. to find turning point.

Minimum point $(-15,-24)$ and maximum point $(-3,\ 0)$.

The graph of $y=\dfrac{x^2+6x+9}{x+9}$.



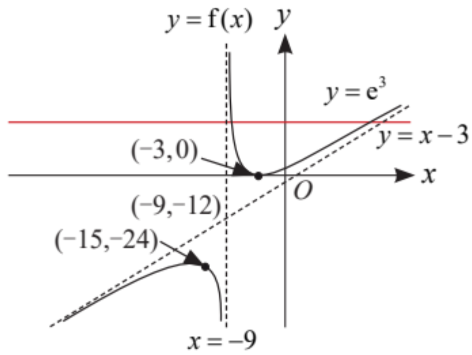
(c) Given $\ln\left|x^2+ax+b\right|-\ln(x+9)\geq 3$,

$$\ln\left(\frac{\left|x^2+6x+9\right|}{x+9}\right)\geq 3$$

$$\ln\left(\frac{\left|x^2+6x+9\right|}{x+9}\right)\geq 3,\ \text{since } x^2+6x+9=(x+3)^2\geq 0 \text{ for all } x\in\mathbb{R}$$

$$\frac{x^2+6x+9}{x+9}\geq e^3$$

Add $y=e^3$ into the diagram in **(b)**.



Using GC to solve $\dfrac{x^2 + 6x + 9}{x + 9} = \mathrm{e}^3$.

The x-coordinate of the point of intersections are -7.8357 and 21.921.

From the graph, for $\dfrac{x^2 + 6x + 9}{x + 9} \geq \mathrm{e}^3$

$\therefore \; -9 < x \leq -7.84$ or $x \geq 22.0$

80
Solution

(a) Given $y=\dfrac{4x^2+4x+1}{1-x}$

$$4x^2+(4+y)x+1-y=0 \text{ .... (1)}$$

For equation (1) to have real value(s) of x, discriminant ≥ 0

$$(4+y)^2-4(4)(1-y)\geq 0$$
$$y(y+24)\geq 0$$
$$y\leq -24 \text{ or } y\geq 0$$

$\{y: y\in\mathbb{R},\ y\leq -24 \text{ or } y\geq 0\}$

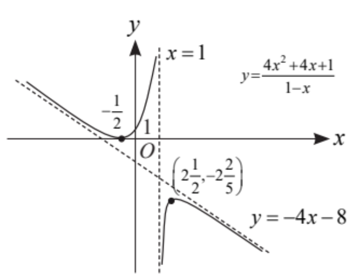
(b) $y=\dfrac{4x^2+4x+1}{1-x}$

$$=-4x-8+\frac{9}{1-x}$$

Equations of asymptotes
$x=1$ is a vertical asymptote.
$y=-4x-8$ is a horizontal asymptote.
Axial intercept : When $x=0$, $y=1$
When $y=0$, $x=-0.5$

Use G.C. to find turning point.
Minimum point $(2.5,-2.4)$ and maximum point $(-0.5,\ 0)$.

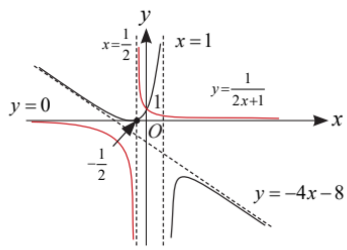
The graph of $y=\dfrac{4x^2+4x+1}{1-x}$.



(c) From the graph, for the line $y = mx - m - 12$ intersects C_1 at two points, the gradient of the line m needs to be flatter.

$\therefore\ m < -4$

(d) Add the graph of $y = \dfrac{1}{2x+1}$ on the same diagram as in part (**b**).



(i) Using graphic calculator to find x-coordinate of the point of intersection.

$\therefore\ x = 0$

To solve $\dfrac{4x^2+4x+1}{1-x} \le \dfrac{1}{2x+1}$ .... (1)

From the graph above, the range of values of x for which $\dfrac{4x^2+4x+1}{1-x} \le \dfrac{1}{2x+1}$ is $-\dfrac{1}{2} < x \le 0$ or $x > 1$. .... (*)

(ii) To solve $\dfrac{4x^2+4|x|+1}{1-|x|} \le \dfrac{1}{2|x|+1}$

Replace $|x|$ in (1): $\dfrac{4x^2+4|x|+1}{1-|x|} \le \dfrac{1}{2|x|+1}$.

Deduce the result in (*). Replace x with ($|x|$) in (*).

$\therefore\ -\dfrac{1}{2} < |x| \le 0$ or $|x| > 1$

$x = 0$ or $x > 1$ or $x < -1$

(e) **Description of sequence of transformations**

The curve C is stretch parallel to the x-axis with a of factor 2 , followed by a stretch parallel to the y-axis with a factor $\dfrac{1}{2}$, followed by reflection in the y-axis.

81
Solution

(a) Let $y = \dfrac{a}{x^2} + bx + c$ ....,...... (1)

Substitute $x = 1.6$ and $y = 2.4$ into (1)

$$-2.4 = \frac{a}{(1.6)^2} + b(1.6) + c \text{ .... } (2)$$

Substitute $x = -0.7$ and $y = 3.6$ into (1)

$$3.6 = \frac{a}{(-0.7)^2} + b(-0.7) + c \text{ .... } (3)$$

Differentiating (1) with respect to x

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-2a}{x^3} + b$$

Substitute $x = 1$ and $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2$ into $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{-2a}{x^3} + b$

$$2 = -2a + b \text{ .... } (4)$$

Using GC to solve (2), (3) and (4) simultaneously

$\therefore\ a = -3.593,\ b = -5.187$ and $c = 7.303$

(b) Substitute $a = -3.593,\ b = -5.187$ and $c = 7.303$ into (1)

$$y = \frac{-3.59345}{x^2} - 5.18691x + 7.30274$$

When $y = 0$,

$$0 = \frac{-3.59345}{x^2} - 5.18691x + 7.30274$$

$$5.18691x^3 - 7.30274x^2 + 3.59345 = 0$$

By GC, $x = -0.589$

(c) Equation of the other asymptote is $y = -5.187x + 7.303$.

82
Solution

Let $x,\ y$ and z be the usual retail price of a packet of cashew nuts, macadamia nuts and almonds respectively.

$$4x+6y+7z=57.05 \text{ .... } (1)$$

Amount of money Jacob is to pay in Supermarket A during the sale:

$$4(0.7)x+4y+7z-10=33.05$$
$$2.8x+4y+7z=43.05 \text{ .... } (2)$$

Amount of money Jacob is to pay in Supermarket B during the sale:

$$4(0.8)x+3y+7(z-0.15)=33.05+5.45$$
$$3.2x+3y+7z=39.55 \text{................} (3)$$

Using GC to solve (1), (2) and (3) : $x=3.5,\ y=4.9,\ z=1.95$

The usual retail price of a packet of cashew nuts, macadamia nuts and almonds is \$3.50, \$4.90 and \$1.95 respectively.

83
Solution

Let the amount of Plus, Power and Ultra in 20 grams of Ultra Power Plus be x, y and z.

Amount of protein contained in each sachet of Ultra Power Plus :

$$\left(\frac{2.6}{20}\right)x+\left(\frac{3.5}{20}\right)y+\left(\frac{2.8}{20}\right)z=3$$

$$0.13x+0.175y+0.14z=3 \text{ .... (1)}$$

Amount of fat contained in each sachet of Ultra Power Plus :

$$\left(\frac{2}{20}\right)x+\left(\frac{1.7}{20}\right)y+\left(\frac{0.5}{20}\right)z=1.5$$

$$0.1x+0.085y+0.025z=3 \text{ .... (2)}$$

Amount of carbohydrate contained in each sachet of Ultra Power Plus :

$$\left(\frac{13.3}{20}\right)x+\left(\frac{12.5}{20}\right)y+\left(\frac{13.9}{20}\right)z=13.0$$

$$0.665x+0.625y+0.695z=13.0 \text{ .... (3)}$$

Using GC to solve (1), (2) and (3) :

$x=6.7419,\ y=8.3089,\ z=4.7821$

To produce 1 kilograms of Ultra Power Plus

Amount of Super Plus $=6.7419\times\left(\frac{1000}{20}\right)=337.095$ grams

Amount of Super Power $=8.3089\times\left(\frac{1000}{20}\right)=415.445$ grams

Amount of Super Ultra $=4.7821\times\left(\frac{1000}{20}\right)=239.105$ grams

84
Solution

Let x, y and z be the number of red, blue and green matchsticks respectively.

$x+y+z=88$ .... (1)

$\frac{z}{6}+x=y$

$6x-6y+z=0$ .... (2)

$\frac{y}{3}+\frac{z}{6}=4\left(\frac{x}{4}\right)$

$6x-2y-z=0$ .... (3)

Using GC, $x=16,\ y=24$ and $z=48$

Alfred has a total of 24 blue matchsticks.

85
Soluton

(a)

	A	B	C
burger meals	x	$3x$	x
chicken rice meals	y	$3y$	$1.5y$
spaghetti meals	z	z	$0.5z$

From the table above,

$$x+y+z=960$$
$$3x+3y+z=2100$$
$$x+1.5y+0.5z=850$$

Using GC, $x=400$, $y=170$ and $z=390$

Total number of chicken rice meals sold from the three vending machines

$=5.5y$

$=5.5(170)$

$=935$

$\therefore$ the total number of chicken rice meals sold from the three vending machines is \$935.

(b) Let the profit of each spaghetti meal sold be a dollars.

Then the profit of each burger meal sold is $1.02a$ and that of chicken rice meal is $1.04a$.

Total profit

$= 5x(1.02a)+5.5y(1.04a)+2.5z(a)$

$= 400(5)1.02a+5.5(170)1.04a+2.5(390)a$

$=3792.4a$

Given that the total monthly profit from the three machines is \$981.10

$\therefore \quad 3792.4a=\$981.10$

$a=\$0.25$

The profit for each spaghetti meal is $\$0.25$.

86
Solution

Let x, y and z be the number of citizens with blood types A, B and O respectively.

$$x + y + z = 1200$$
$$0.35x + 0.4y + 0.45z = 520$$

Using GC, solving in terms of z.

$$x = -800 + z$$
$$y = 2000 - 2z$$

Since, x, y and z are all non - negative integers,

i.e. $x \geq 0$

$-800 + z \geq 0$

$z \geq 800$

Also, $y \geq 0$

$2000 - 2z \geq 0$

$z \leq 1000$

$\therefore \quad 800 \leq z \leq 1000.$